

Optimization Problems

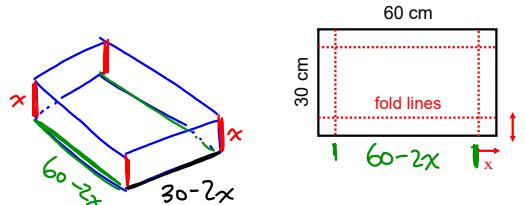
March 20/2018

Optimization:

To realize the best possible outcome, subject to a set of restrictions.

Mathematically, this usually refers to a maximum or minimum, which can be identified through the use of calculus.

Ex.1 A piece of sheet metal, 60 cm by 30 cm, will be used to make a box with an open top. Determine the dimensions that will maximize volume.



$$V = lwh$$

$$V(x) = x(60-2x)(30-2x)$$

for CV, need $V'(x)$

- * ① expand fully & take derivative
 - ② expand pair & use product rule
 - ③ use product rule for multiple (3) terms
- $$V(x) = f \cdot g \cdot h$$
- $$V'(x) = f'gh + fg'h + fgh'$$

best in this case

$$\begin{aligned} V(x) &= x(60-2x)(30-2x) \\ &= x(1800 - 180x + 4x^2) \\ &= 1800x - 180x^2 + 4x^3 \end{aligned}$$

$$V'(x) = 1800 - 360x + 12x^2$$

CV: set $V'(x) = 0$

$$\begin{aligned} 0 &= 1800 - 360x + 12x^2 \quad \div 12 \\ 0 &= 150 - 30x + x^2 \end{aligned}$$

$$QF: x = 15 \pm 5\sqrt{3}$$

$$\begin{array}{ll} x = 15 + 5\sqrt{3} & x = 15 - 5\sqrt{3} \\ \approx 23.66 & \approx 6.34 \end{array}$$

consider $l w h$
 $x \swarrow \downarrow \searrow 60-2x \quad 30-2x$
 $30-2(23.66) < 0$ invalid
 reject $x = 23.66$

$$\begin{array}{ll} CV: x = 15 - 5\sqrt{3} & \\ & \approx 6.34 \end{array}$$

Max or min?

$$CV: x = 15 - 5\sqrt{3}$$

classify by comparing $\underset{?}{\text{vs end points}}$

$$V = lwh$$

$$l > 0 \quad w > 0 \quad h > 0$$

$$x > 0 \quad 60-2x > 0 \quad 30-2x > 0$$

$$60 > 2x \quad 30 > 2x$$

$$30 > x$$

$$x < 30$$

$$15 > x$$

$$x < 15$$

$$x \in (0, 15)$$

$$\lim_{x \rightarrow 0} V(x) =$$

$$\lim_{x \rightarrow 15} V(x) =$$

Strategy for solving optimization problems:

1. Read the problem carefully. Determine a function of the independent variable that represents the quantity to be optimized (the dependent variable).
2. Draw a diagram (if possible).
3. Determine the domain of the function.
4. Compare all extreme values and end points to find absolute maximum or minimum values.
5. Answer the original problem.

Assigned Work:

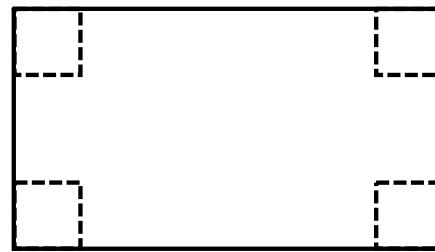
p.145 # 1, 3-12

Assigned Work:

p.145 # 1, 3-12

12
10

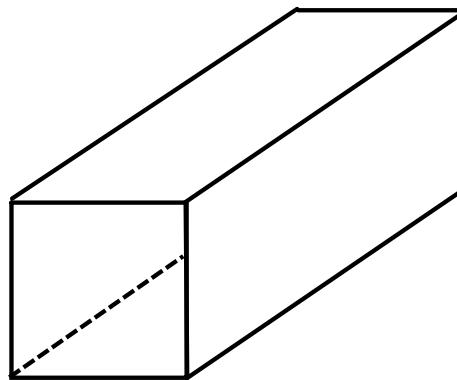
Ex1: p.145 #4



Ex2: p.145 #7



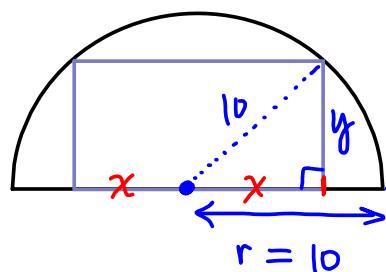
Ex3: p.145 #8



Ex4: p.146 #10

$$A_{\text{rect}} = l \omega$$

\downarrow
 $2x$ $\sqrt{100 - x^2}$



$$A(x) = 2x\sqrt{100 - x^2}$$

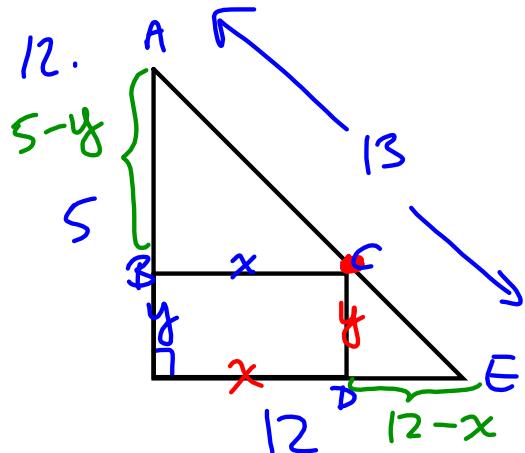
$$A'(x) = \underline{\hspace{2cm}}$$

$$\text{set } A'(x) = 0$$

solve

test end points
 $x \in (0, 10)$

$$\begin{aligned}
 x^2 + y^2 &= 10^2 \\
 y^2 &= 100 - x^2 \\
 y &= \pm \sqrt{100 - x^2} \\
 \text{reject } y < 0 & \\
 (\text{y is a length})
 \end{aligned}$$



$\triangle ABC \sim \triangle CDE$ (AA~)

$$\frac{x}{12} = \frac{5-y}{5}$$

$$5x = 60 - 12y$$

$$12y = 60 - 5x$$

$$y = \frac{60 - 5x}{12}$$

$$A = xy$$

$$A(x) = x \left(\frac{60 - 5x}{12} \right)$$