Optimizing Distance

March 21/2018

Optimization:

To realize the best possible outcome, subject to a set of restrictions.

Mathematically, this usually refers to a maximum or minimum, which can be identified through the use of calculus.

Assigned Work:

p.147 # 15, 16, 20
Handout # 1-5, 8, 9

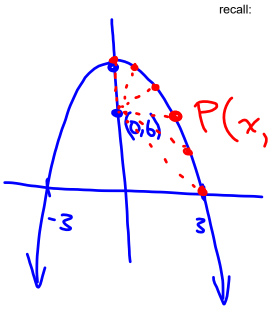
Ex.1 Find the minimal distance from the point (0, 6) to the curve $y = 9 - x^2$. Discuss how to find the coordinate of the point on the curve that is closest to (0, 6).

$y = -x^2 + 9$

recall: The distance formula, which is simply the Pythagorean theorem applied between two points.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

sketch:



$P(x, 9-x^2)$

$P_1(0,6)$ $P_2(x, 9-x^2)$

$$d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$

$$d = \sqrt{(x-0)^2 + (9-x^2-6)^2}$$

$$d(x) = \sqrt{x^2 + (3-x^2)^2}$$

① $d'(x)$

② simplify, $d'(x)$

$$= \sqrt{x^2 + 9 - 6x^2 + x^4}$$

$$= (x^4 - 5x^2 + 9)^{\frac{1}{2}}$$

$d(x) = (x^4-5x^2+9)^{\frac{1}{2}}$

$d'(x) = \frac{1}{2}(x^4-5x^2+9)^{-\frac{1}{2}}(4x^3-10x)$

$= \frac{2x(2x^2-5)}{2\sqrt{x^4-5x^2+9}}$

CV: set $d'(x)=0$

$x=0$, $2x^2-5=0$

$x=\pm\sqrt{\frac{5}{2}}$

* from graph, it is obvious there are no restrictions on x

classify: $x \in (-\infty,\infty)$

$\lim_{x\to-\infty} d(x) = \infty$ $\lim_{x\to\infty} d(x) = \infty$

$d(-\sqrt{\frac{5}{2}}) \doteq 1.66$ min $d(0) = 3$ $d(\sqrt{\frac{5}{2}}) \doteq 1.66$ min

$y = 9-x^2$ sub $\sqrt{\frac{5}{2}}$

$y = 9-\frac{5}{2}$

$= \frac{13}{2}$

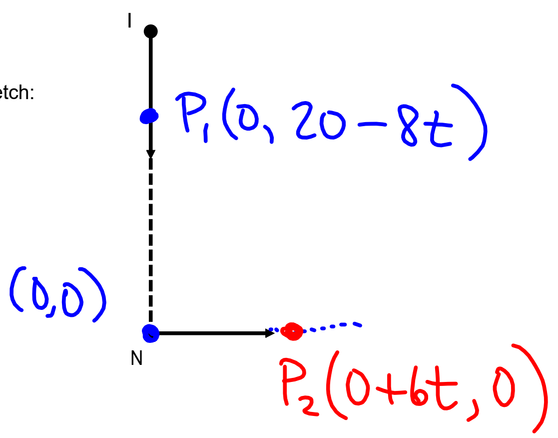
$\therefore$ the min distance occurs at the points $(\sqrt{\frac{5}{2}}, \frac{13}{2})$ and $(-\sqrt{\frac{5}{2}}, \frac{13}{2})$

Ex.2 (see p.143 Exercise # 3)

Ian's house is located 20 km north of Nada's house. One morning, Ian leaves his house and jogs south at a 8km/h. At the same time, Nada leaves her house and jogs east at 6km/h.

If they both run for 2.5 hours, when will Ian and Nada be closest together?

sketch:



$$d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

Ex.3 (see p.147 #15)

A train leaves the station at 10:00 a.m. and travels due south at 60 km/h. Another train has been heading due west at 45 km/h and reaches the same station at 11:00 a.m.

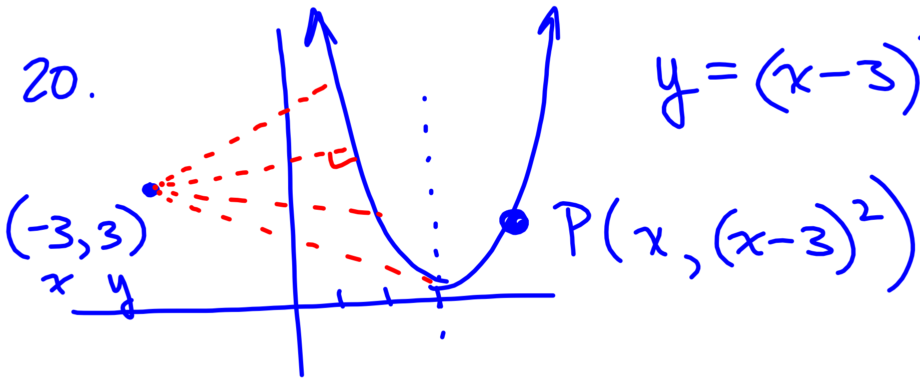
At what time of day (in hours and minutes) were the two trains closest to each other?

sketch:

Assigned Work:

p.147 # 15, 16, 20
Handout # 1-5, 8, 9
4

20.

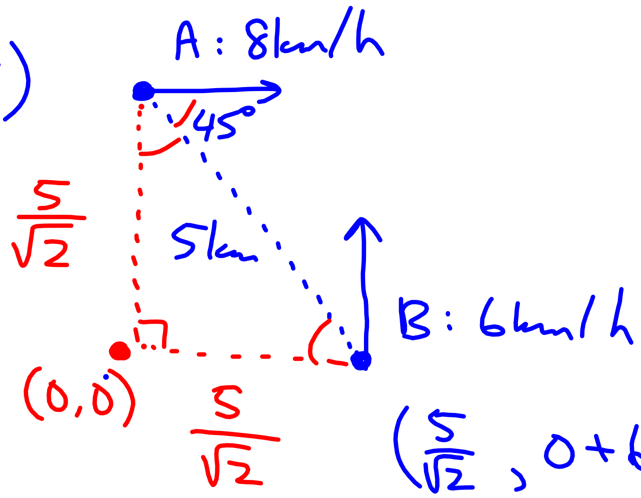


$y=(x-3)^2$

$$d=\sqrt{(x-(-3))^2+((x-3)^2-3)^2}$$

WS #4.

$\left(0+8t, \frac{5}{\sqrt{2}}\right)$
$\quad x_1 \qquad y_1$



$\left(\frac{5}{\sqrt{2}}, 0+6t\right)$
$\quad x_2 \qquad y_2$