Optimizing Time

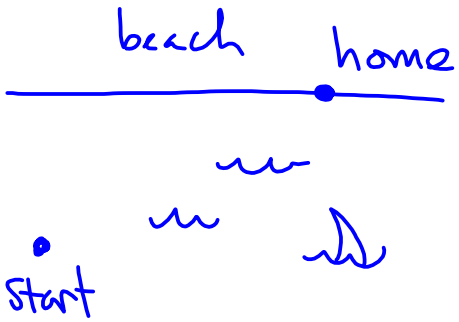
March 22/2018

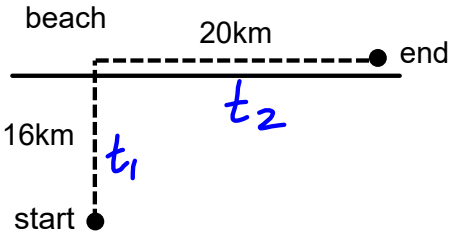
Ex.1 An amphibious vehicle is in the ocean, 16 km away from the beach. The operator's home is 20 km down the shore. To get back home, the vehicle can travel at a speed of 30 km/h in water and 70 km/h on land.

a) Consider some simple paths they could take to get home, and the time required for each path.

b) Determine the route taken to minimize their travel time.

Hint: A neat, labelled diagram will help visualize and solve problem.



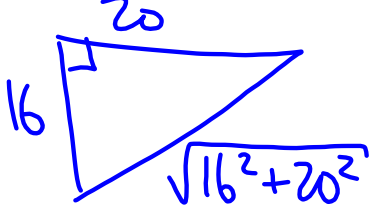


Recall: $d = v \times t$

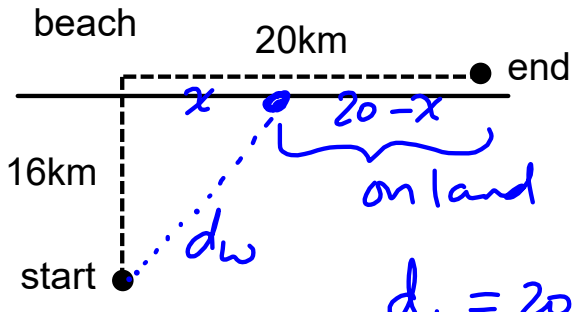
Consider some simple scenarios
(1) straight to beach, then home
(2) straight home (diagonal)

$t_1 = \frac{16}{30}$ $t_2 = \frac{20}{70}$ $t = \frac{d}{v}$

$t = t_1 + t_2$



$t = \frac{\sqrt{16^2+20^2}}{30}$



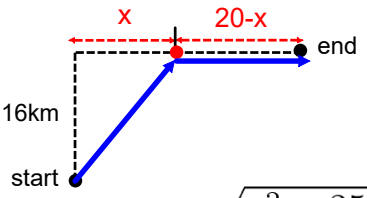
Recall: $d = v \times t$

What about landing somewhere along the beach?

$d_L = 20 - x$

$d_w = \sqrt{x^2 + 16^2}$

$t_w = \frac{\sqrt{x^2+16^2}}{30}$ $t_L = \frac{20-x}{70}$



Recall: $d = v \times t$

What about landing somewhere along the beach?

$$t(x) = \underbrace{\frac{\sqrt{x^2+256}}{30}}_{t_w} + \underbrace{\frac{20-x}{70}}_{t_L}$$

$t(x) = \frac{1}{30}(x^2+256)^{\frac{1}{2}} + \frac{1}{70}(20-x)$

$t'(x) = \frac{1}{30}\left[\frac{1}{2}(x^2+256)^{-\frac{1}{2}}(2x)\right] + \frac{1}{70}(-1)$

Set $t'(x) = 0$

$0 = \frac{x}{30\sqrt{x^2+256}} - \frac{1}{70}$

$\frac{1}{70} = \frac{x}{30\sqrt{x^2+256}}$

$(30\sqrt{x^2+256})^2 = (70x)^2$

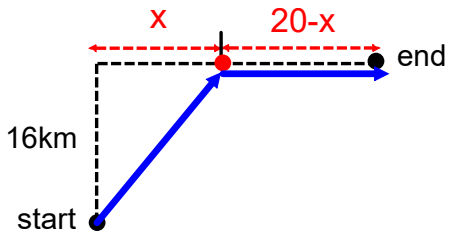
$9(x^2+256) = 49x^2$

$9x^2 + 2304 = 49x^2$

$2304 = 40x^2$

$\pm\sqrt{\frac{2304}{40}} = x$

reject $x < 0$



Recall: $d = v \times t$

Want minimum time (absolute minimum), so check local extrema and end points

end points: $x \in [0, 20]$

$x = \sqrt{\frac{2304}{40}}$

$\doteq 7.6$

$t(0)$

$\doteq 0.819$

$t\left(\sqrt{\frac{2304}{40}}\right)$

$\doteq 0.768$

absolute min.

$t(20)$

$\doteq 0.854$

∴ the landing should be 7.6 km along the beach (or 12.4 km away from home)

Assigned Work:

Handout #11, 12, 13, 14, 15