Exponential & Logarithmic Optimization + curve sketching

Strategy for solving optimization problems:

1. Draw a diagram (if possible).
2. Identify an equation in a single variable.
3. Domain of function.
4. Identify absolute max or min from extrema, end points.
5. Answer the problem.

For exponential functions, also consider:

(a) Rules for changing exponential expressions.
(b) Take log of both sides to isolate exponents.
(c) Limit behaviour & asymptotes.

$\pm\infty$

$5^x = 10$

Ex.1 Use the curve sketching algorithm to sketch

$$y = x^2 e^{2x}$$

(1) 1st and 2nd derivatives

$y = f \cdot g \cdot h$

$y' = f'gh + fg'h + fgh'$

$y' = 2x e^{2x} + x^2 e^{2x} \cdot (2)$

$= 2x e^{2x} + 2x^2 e^{2x}$

$= 2x e^{2x}(1+x)$

$y'' = 2e^{2x}(1+x) + 2x e^{2x}(2)(1+x) + 2x e^{2x}(1)$

$= 2e^{2x}(1+x) + 4x e^{2x}(1+x) + 2x e^{2x}$

$= 2e^{2x}\left[(1+x) + 2x(1+x) + x\right]$

$= 2e^{2x}\left[1 + x + 2x + 2x^2 + x\right]$

$= 2e^{2x}\left[2x^2 + 4x + 1\right]$

Ex.1 Use the curve sketching algorithm to sketch

$$y = x^2 e^{2x}$$

(2) critical values

$$y' = 2x(1+x)e^{2x} \quad \text{set } y' = 0$$

$$y'' = 2(2x^2+4x+1)e^{2x} \quad \text{set } y'' = 0$$

$y' = 0$

$0 = 2x(1+x)e^{2x}$

$2x = 0 \quad x = 0$

$1+x = 0 \quad x = -1$

$e^{2x} = 0$ no sol.

$y'' = 0$

$0 = 2(2x^2+4x+1)e^{2x}$

$2x^2+4x+1 = 0$

$b^2-4ac = 4^2-4(2)(1) = 8$ 2 sol

$x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x = \dfrac{-4 \pm \sqrt{8}}{4}$ $\quad \sqrt{8} = \sqrt{4\cdot 2} = 2\sqrt{2}$

$= -1 \pm \dfrac{2\sqrt{2}}{4}$

$x = -1 \pm \dfrac{\sqrt{2}}{2}$

$x \doteq -0.293 \quad x \doteq -1.707$

Ex.1 Use the curve sketching algorithm to sketch

$$y = x^2 e^{2x}$$

(3) 2nd derivative test

$$0 = 2x(1+x)e^{2x} \rightarrow x = 0, x = -1 \ ?$$

$$0 = 2(2x^2+4x+1)e^{2x} \rightarrow x \approx -0.293, x \approx -1.707$$

$-2 \quad -1.7 \quad -1 \quad -0.3 \quad 0$

	$(-\infty, -1.7)$	$(-1.7, -0.3)$	$(-0.3, \infty)$
2	+	+	+
$2x^2+4x+1$	$8-8+1=1$ +	$2-4+1=-1$ −	+
e^{2x}	+	+	+
	+	−	+
	CU	CD	CU
		$x=-1$ max	$x=0$ min

Ex.1 Use the curve sketching algorithm to sketch

$$y = x^2 e^{2x}$$

(4) end behaviour

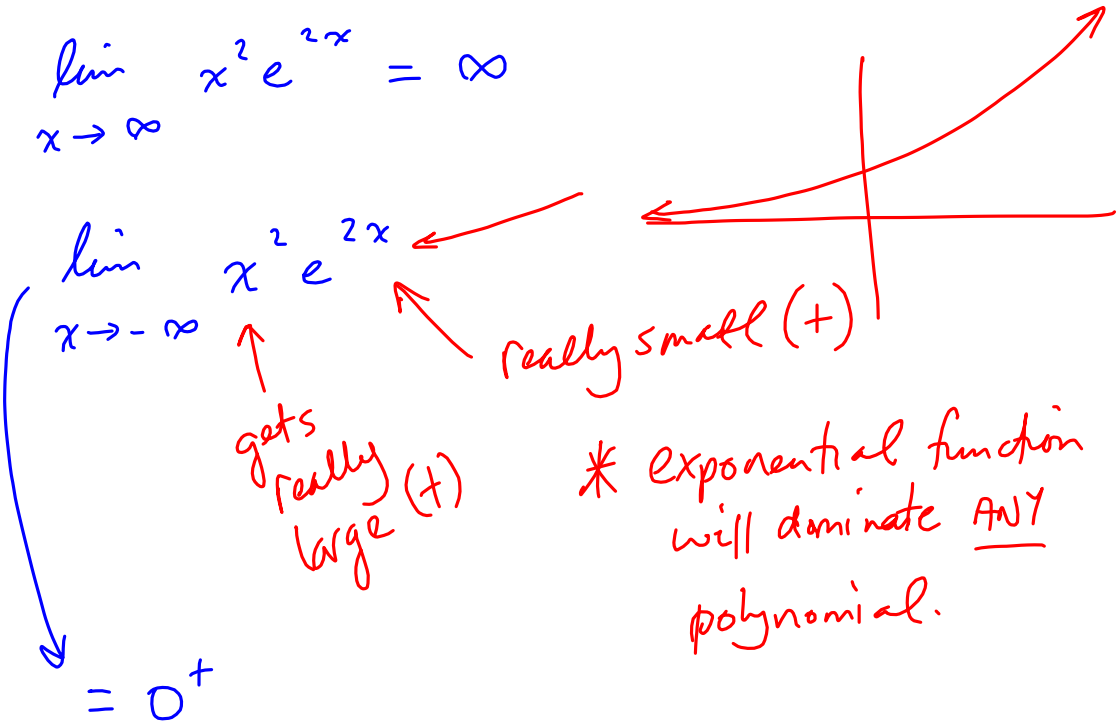
$\lim_{x \to \infty} x^2 e^{2x} = \infty$

$\lim_{x \to -\infty} x^2 e^{2x}$ = really small (+)

gets really large (+)

$= 0^+$

* exponential function will dominate ANY polynomial.



Ex.1 Use the curve sketching algorithm to sketch

$$y = x^2 e^{2x}$$

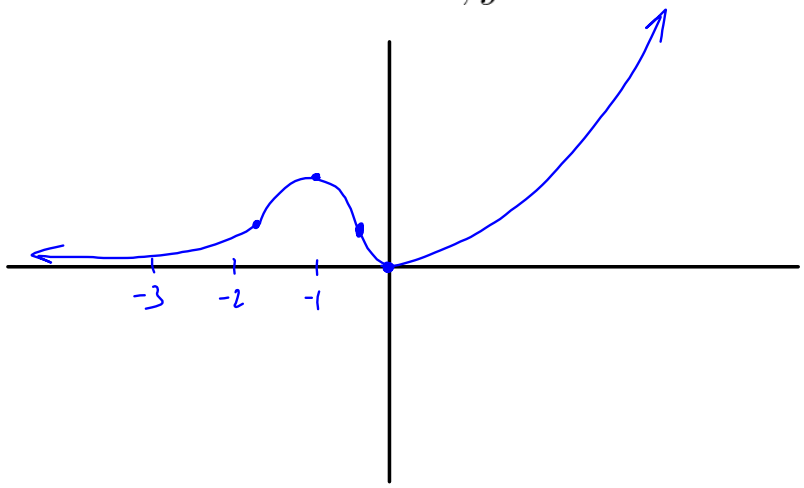
(5) sketch graph

$$\min : (0,0) \quad \max : \left(-1, \frac{1}{e^2}\right) \approx (-1, 0.135)$$

$$\text{POI} : (-0.3, 0.05), (-1.7, 0.095)$$

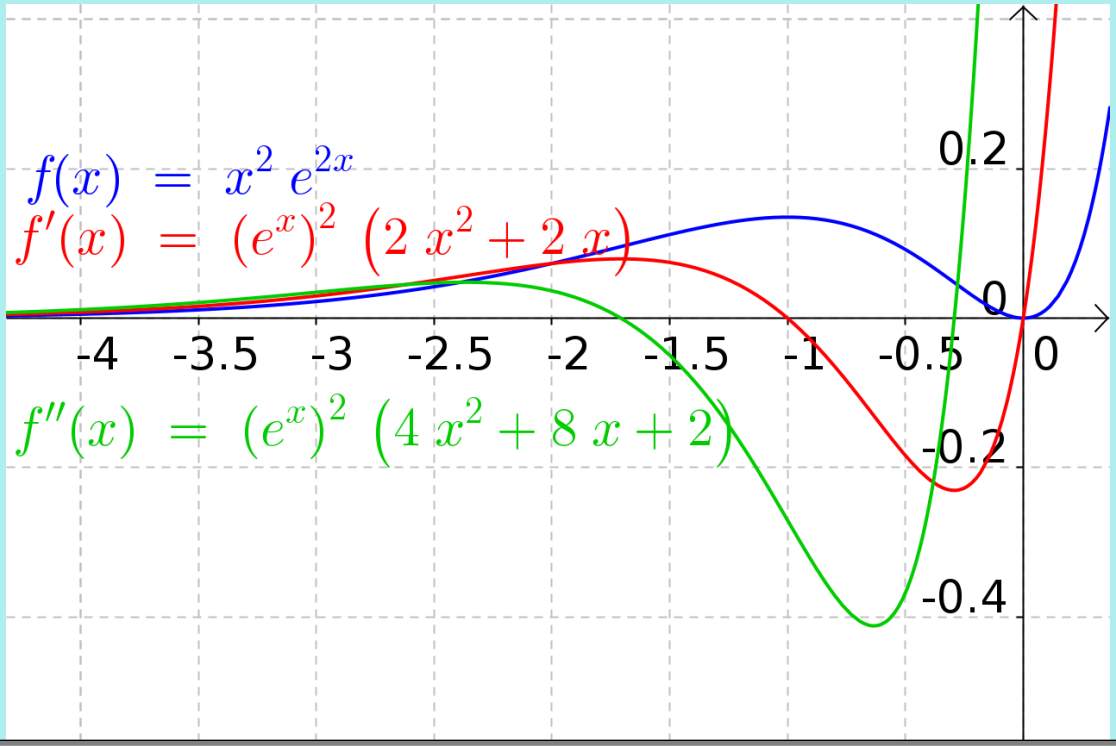
end behaviour: as $x \to \infty, y \to \infty$

as $x \to -\infty, y \to 0^+$



Ex.1 Use the curve sketching algorithm to graph

$$y = x^2 e^{2x}$$



Assigned Work:

Read examples 1 & 2, p.241-244

p. 245 # 3, 4, 5, 6, 8, 9, 12bc