Derivative of Logarithmic Functions
(see p.571 - 582)

From our previous lessons on exponential functions:

$$f(x) = e^x$$
$$f'(x) = e^x$$

$$f(x) = e^{g(x)}$$
$$f'(x) = e^{g(x)}g'(x)$$

$$f(x) = b^x$$
$$f'(x) = \ln(b)b^x$$

$$f(x) = b^{g(x)}$$
$$f'(x) = \ln(\mathrm{b})\mathrm{b}^{g(x)}\,g'(x)$$

Derivative of Logarithmic Functions
(see p.571 - 582)

Recall: An exponential function, $f(x) = b^x$

has an inverse, $f^{-1}(x) = \log_b(x)$

(from Advanced Functions).

Similarly, the natural exponential function, $f(x) = e^x$
has an inverse which is the natural logarithm,

$$f^{-1}(x) = \log_e(x) = \ln(x)$$

"lawn"

Recall: Laws of Logarithms

$$\log_b(xy) = \log_b(x) + \log_b(y)$$

$$\log_b\left(\frac{x}{y}\right) = \log_b(x) - \log_b(y) \qquad \log_b x = \frac{\log_a x}{\log_a b}$$

$$\log_b(x^y) = y\log_b(x)$$

For the natural logarithm, they become:

$$\ln(xy) = \ln(x) + \ln(y)$$

$$\ln\left(\frac{x}{y}\right) = \ln(x) - \ln(y)$$

$$\ln(x^y) = y\ln(x)$$

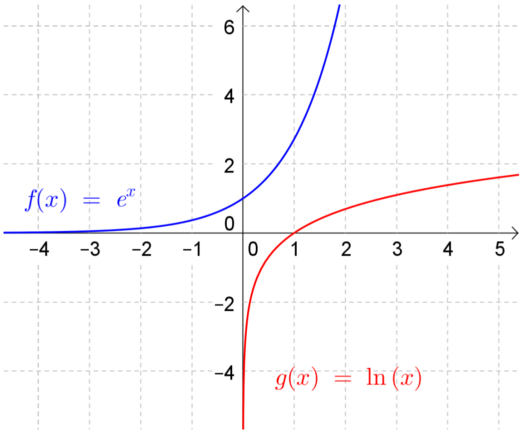
Given $f(x) = \ln x$, what is the derivative?

As we did with exponential functions, we will use graphing technology to look at the slope of the function, and look for a pattern.

see Geogebra demo: lnx Slope Demo

Given $f(x) = \ln(x)$, $x > 0$

$$f'(x) = \frac{1}{x},\ x > 0$$



Applying the chain rule:

$$f(x) = \ln\left[g(x)\right]$$

argument > 0

$$f'(x) = \frac{1}{g(x)} g'(x),\ g(x) > 0$$

Ex.1 Determine the derivative for

a) $y = \ln(5x)$ b) $y = \ln(7x^2)$

c) $y = \ln\sqrt{5x}$ d) $y = 3^x \ln(x^3)$

(a) $y' = \frac{1}{\cancel{5}x} \cdot (\cancel{5})$

$= \frac{1}{x}$

(b) $y' = \frac{1}{\cancel{7}x^{\cancel{2}\,1}} \cdot (\cancel{14}^{\,2}\cancel{x})$

$= \frac{2}{x}$

(c) $y = \ln(5x)^{\frac{1}{2}}$

$= \frac{1}{(5x)^{\frac{1}{2}}} \cdot \left(\frac{1}{2}\right)(5x)^{-\frac{1}{2}}(5)$

$= \frac{\cancel{5}^{\,1}}{2(\cancel{5}x)^1}$

$= \frac{1}{2x}$

or

$y = \frac{1}{2}\ln(5x)$

$y' = \frac{1}{2}\left[\frac{1}{\cancel{5}x} \cdot (\cancel{5})\right]$

$= \frac{1}{2x}$

Ex.1 Determine the derivative for

a) $y = \ln(5x)$

b) $y = \ln(7x^2)$

c) $y = \ln\sqrt{5x}$

d) $y = 3^x \ln(x^3)$

(d) $y = 3^x(3 \ln x)$

$y = 3(3^x \ln x)$

$y' = 3\left[3^x \ln 3 \ln x + 3^x \cdot \frac{1}{x}\right]$

$= 3 \cdot 3^x\left[\ln 3 \ln x + \frac{1}{x}\right]$

Given $f(x) = \log_b(x)$

$$f'(x) = \left(\frac{1}{x}\right)\left(\frac{1}{\ln(b)}\right) = \frac{1}{x \ln b}$$

Applying the chain rule:

$$f(x) = \log_b\left[g(x)\right]$$

$$f'(x) = \left(\frac{1}{g(x)}\right)\underbrace{g'(x)}_{1}\left(\frac{1}{\ln(b)}\right) = \frac{g'(x)}{g(x)\ln b}$$

Ex.2 Find the derivative for

a) $y = \log_3(6x)$ b) $y = \log\sqrt[3]{x^4}$

c) $y = \sqrt{\log x}$ d) $y = \dfrac{\log_2 4x^3}{e^{5x}}$

(a) $y' = \dfrac{1}{6x \ln 3} \cdot (6)$

$= \dfrac{1}{x \ln 3}$ ✓

$= \dfrac{1}{(\ln 3)(x)}$ ✓

(b) $y = \log_{10} x^{\frac{4}{3}}$

$y = \frac{4}{3} \log_{10} x$

$y' = \frac{4}{3}\left[\dfrac{1}{x \ln 10}\right]$

$= \dfrac{4}{3x \ln 10}$ ✓

Ex.2 Find the derivative for

a) $y = \log_3(6x)$ b) $y = \log\sqrt[3]{x^4}$

c) $y = \sqrt{\log x}$ d)

(c) $y = (\log_{10} x)^{\frac{1}{2}}$

$y' = \frac{1}{2}(\log_{10} x)^{-\frac{1}{2}} \cdot \dfrac{1}{x \ln 10}$

$= \dfrac{1}{2x \ln 10 (\log x)^{\frac{1}{2}}}$

$y = \dfrac{\log_2 4x^3}{e^{5x}} \longrightarrow \log_2(4x^3)$

$y' = \dfrac{\dfrac{1}{\cancel{4x^3} \ln 2} \cdot (\cancel{12x^2}) \cdot e^{5x} - \log_2(4x^3) \cdot e^{5x} \cdot (5)}{(e^{5x})^2}$

$= \dfrac{\cancel{(e^{5x})}\left[\dfrac{3}{x \ln 2} - 5 \log_2 4x^3\right]}{(e^{5x})^{2-1}}$

$y' = \dfrac{\dfrac{3}{x \ln 2} - 5 \log_2 4x^3}{e^{5x}}$

$$y = \frac{\log_2 4x^3}{e^{5x}} \longrightarrow y = \frac{f(x)}{g(x)}$$

$$y = \frac{f'g - fg'}{g^2}$$

$$f'(x) = \frac{1}{\ln 2}\left(\frac{1}{\cancel{4}x^{\cancel{3}}}\right)\cdot\left(\cancel{12}^{3}\cancel{x^2}\right)$$

$$= \frac{3}{x\ln 2}$$

$$g'(x) = 5e^{5x}$$

$$y' = \frac{\frac{3}{x\ln 2}\cdot e^{5x} - (\log_2 4x^3)(5e^{5x})}{(e^{5x})^2}$$

$$= \frac{\cancel{(e^{5x})}\left[\frac{3}{x\ln 2} - 5\log_2 4x^3\right]}{(e^{5x})^{\cancel{2}}}$$

$$= \frac{\frac{3}{x\ln 2} - 5\log_2 4x^3}{e^{5x}}$$

$$= \frac{\frac{1}{x\ln 2}\left[3 - 5x\ln 2\log_2 4x^3\right]}{e^{5x}}$$

$$= \frac{3 - 5x\ln 2\log_2 4x^3}{x\ln 2\, e^{5x}}$$

Assigned Work:

p.575 # 3 - 13 (Omit 5c, 9bc)

p.578 # 1 - 5, 7, 9, 11

$$y = \underbrace{x^5}_{f}\underbrace{e^{x^2}}_{g}$$

$$y' = f'g + fg'$$

$$f' = 5x^4$$

$$g(x) = e^{x^2}$$

$$= e^{h(x)}$$

$$g'(x) = e^{h(x)}\cdot h'(x)$$

$$= e^{x^2}(2x)$$

$$= 2xe^{x^2}$$

$$y' = 5x^4e^{x^2} + x^5(2xe^{x^2})$$

$$= 5x^4e^{x^2} + 2x^6e^{x^2}$$

$$= x^4e^{x^2}(5 + 2x^2)$$

Attachments

Deriv of e^x Demo.ggb