Recall: Given $f(x) = b^x$

$$f'(x) = b^x \lim_{h \to 0} \frac{b^h - 1}{h}$$

$y = e^x$

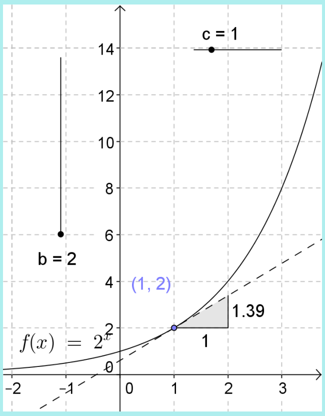
Notice that the limit does not depend on x, only b.
By taking the ratio,

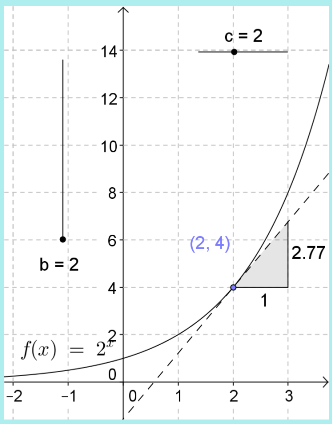
$$\frac{f'(x)}{f(x)} = \lim_{h \to 0} \frac{b^h - 1}{h}$$

we are able to isolate the limit. Now we consider this ratio from available data (graphing technology).

Let $f(x) = 2^x$

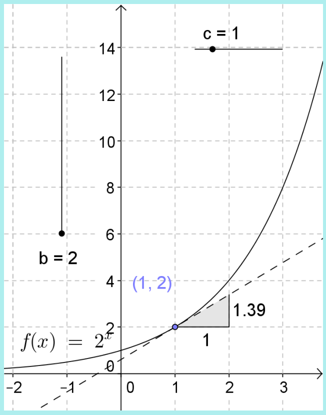
x	$f(x)$	$f'(x)$	$\frac{f'(x)}{f(x)}$
0	1	0.69	$\frac{0.69}{1} = 0.69$
1	2	1.39	
2	4	2.77	
3			

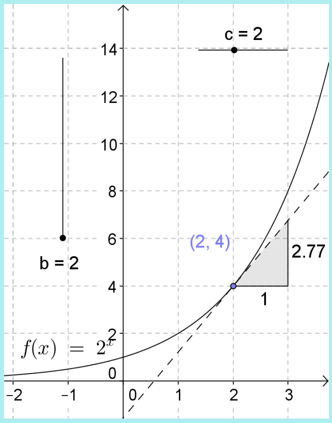




Let $f(x) = 2^x$

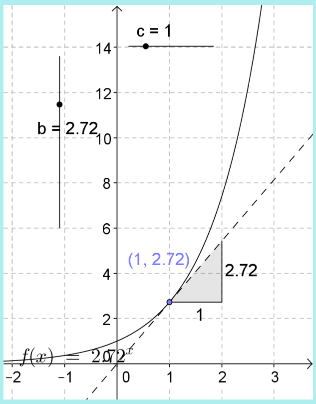
x	$f(x)$	$f'(x)$	$\dfrac{f'(x)}{f(x)}$
0	1	0.69	0.69
1	2	1.39	0.695
2	4	2.77	0.6925
3	8	5.55	0.69375

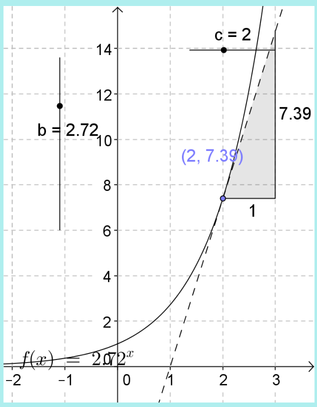




Let $f(x) = e^x$

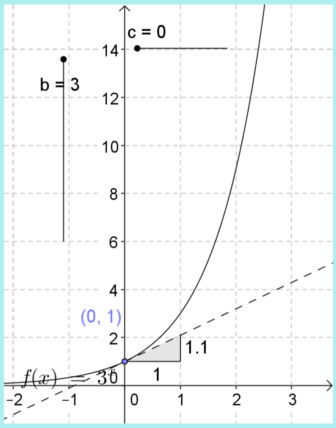
x	$f(x)$	$f'(x)$	$\dfrac{f'(x)}{f(x)}$
0	1	1	1
1	e	2.72	1
2	e^2	7.39	1
3	e^3	20.09	1

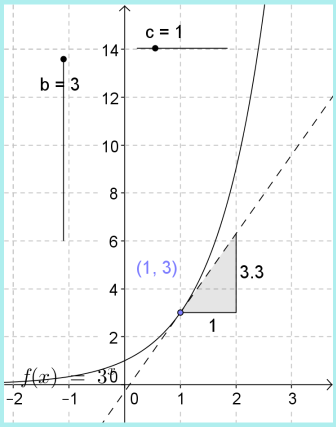




Let $f(x) = 3^x$

x	$f(x)$	$f'(x)$	$\frac{f'(x)}{f(x)}$
0	1	1.1	1.1
1	3	3.3	1.1
2	9	9.89	1.1
3	27	29.66	1.1





Can we represent this ratio with a function?

Is there a function g(x), such that:

$$\lim_{h \to 0} \frac{b^h - 1}{h}$$

$$g(2) \doteq 0.69$$

$$g(e) = 1$$

$$g(3) \doteq 1.10$$

Focus on the result with 'e'. What function, with an input of 'e', would yield 1?

Consider the question another way: What function, with an input of 1, would yield 'e'?

$$g(e) = 1$$

Focus on the result with 'e'. What function, with an input of 'e', would yield 1? inverse

Consider the question another way: What function, with an input of 1, would yield 'e'? ?

$$f(1) = e$$

$f(x) = e^x$

$f(1) = e^1$

$= e$

$y = e^x$

for inverse

$x = \log_e y$

swap x, y

$y = \log_e x$

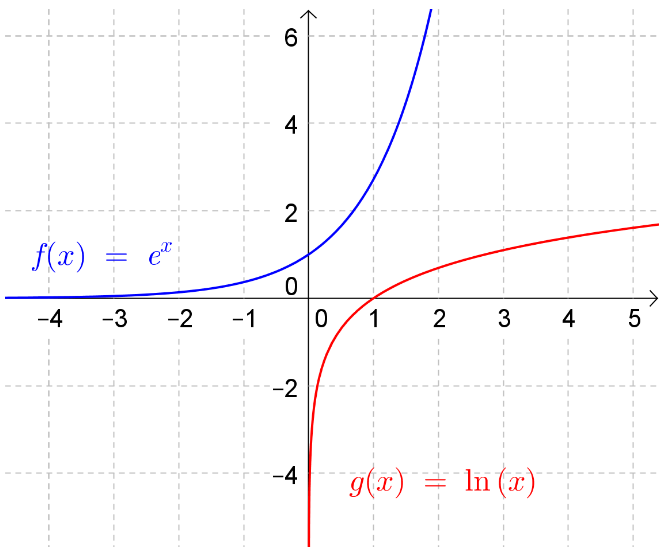
$g(x) = \log_e x$

$g(x) = \underbrace{\ln}_{\text{natural log.}} x$

Derivative of General Exponential Functions

Consider $f(x) = e^x$, where 'e' is Euler's number, $e \doteq 2.718$.

From the graph, it is clearly a function, which means it must have an inverse (which turns out to also be a function).



The inverse is

$$g(x) = \ln(x)$$

(pronounced "lawn") which is the natural logarithm.

It can also be written

$$g(x) = \log_e(x)$$

Given $f(x) = b^x$, we used limits to determine

$$f'(x) = b^x \lim_{h \to 0} \frac{b^h - 1}{h}$$

By using graphs and tangent lines, we can show (numerically) that:

$$\lim_{h \to 0} \frac{b^h - 1}{h} = \log_e(b) = \ln(b)$$

Therefore, given

chain rule

$$\boxed{\begin{aligned} f(x) &= b^x \\ f'(x) &= \ln(b)b^x \end{aligned}} \qquad \boxed{\begin{aligned} f(x) &= b^{g(x)} \\ f'(x) &= \ln(b)b^{g(x)}g'(x) \end{aligned}}$$

Ex. Determine the derivative of each.

(a) $f(x) = 7^x$ (b) $y = 5^{\sqrt{x}}$ (c)

$f'(x) = (\ln 7)(7^x)$ ✓

$\cancel{= \ln 7\ 7^x}$?

$\cancel{\ln 7 7^x}$?

$\cancel{\ln (7 \cdot 7^x)}$?

$y' = (\ln 5)(5^{\sqrt{x}})\left(\frac{1}{2\sqrt{x}}\right)$

$\sqrt{x} = x^{\frac{1}{2}}$

$\frac{d}{dx}\left(x^{\frac{1}{2}}\right) = \frac{1}{2}x^{-\frac{1}{2}}$

$= \frac{1}{2\sqrt{x}}$

$y' = \frac{(\ln 5)(5^{\sqrt{x}})}{2\sqrt{x}}$

$g(x) = \frac{e^{x^2}}{3^x}$

$g'(x) = \frac{(\cancel{\ln e})(e^{x^2})(2x)(3^x) - e^{x^2}(\ln 3)(3^x)}{(3^x)^2}$ 1

$= \frac{2x e^{x^2} 3^x - \ln 3\, e^{x^2} 3^x}{3^{2x}}$

∴ simplify

Assigned Work:

p.240 # 1, 2, 4, 5, 6, 9

Attachments

Deriv of e^x Demo.ggb