UNIT 4

Derivatives of Exponential Functions, Logarithmic Functions, and Trigonometric Functions

Consider the exponential function:

$$f(x) = b^x, \quad b \in (0,1) \cup (1, \infty)$$

How would we determine the slope of the tangent, or derivative? None of our rules currently apply.

$f'(x)$?

~~power rule?~~

$$\cancel{f(x) = b^x}$$

$$\cancel{f'(x) = x b^{x-1}}$$

for power rule,

x^n

indep. var

$f(x) = b^x$

constant

first principles

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{b^{x+h} - b^x}{h} \qquad CF = b^x$$

$$= \lim_{h \to 0} \frac{b^x b^h - b^x}{h}$$

$$= \lim_{h \to 0} \frac{b^x (b^h - 1)}{h}$$

$$= b^x \underbrace{\left[\lim_{h \to 0} \frac{b^h - 1}{h} \right]}_{?}$$

$$f(x) = b^x$$

$$f'(x) = b^x \lim_{h \to 0} \frac{b^h - 1}{h}$$

It would be convenient if there were a value for b where

$$\lim_{h \to 0} \frac{b^h - 1}{h} = 1$$

because then we would have: $f(x) = b^x$

$$f'(x) = b^x$$

see Geogebra: "b^x Slope Demo"

It would be convenient if there were a value for b where

$$\lim_{h \to 0} \frac{b^h - 1}{h} = 1$$

There is such a value (not proven here), b = 2.71828...

The value is known as Euler's number, 'e'.
(pronounced "oiler's number")

$$e \doteq 2.71828$$

where $f(x) = e^x$ and $f'(x) = e^x$

Derivative of THE Exponential Function

Given $f(x) = b^x$, we can use limits to show that

$$f'(x) = b^x \lim_{h\to 0} \frac{b^h - 1}{h}$$

There is one value for *b*, *b* = *e*, that yields

$$\lim_{h\to 0} \frac{e^h - 1}{h} = 1$$

The value is known as Euler's number, 'e', which is a non-repeating irrational number (like pi).
(pronounced "oiler's number")

$$e \doteq 2.71828$$

Therefore, given $f(x) = e^x$
$f'(x) = e^x$

because $\frac{d}{dx}(x) = 1$

We can also apply other derivative rules, with the chain rule used frequently.

$$f(x) = e^{g(x)} \qquad f'(x) = e^{g(x)}g'(x)$$

Ex.1 Find the derivative of each.

(a) $y = -3e^{5x}$

$y = -3e^{g(x)}$, $g(x) = 5x$

$y' = -3 \cdot e^{5x}(5)$

$y' = -15e^{5x}$

(b) $y = \underbrace{x^5}\underbrace{e^{x^2}}$

$y' = 5x^4 e^{x^2} + x^5 \underbrace{e^{x^2}(2x)}$

$= 5x^4 e^{x^2} + 2x^6 e^{x^2}$

let $f(x) = e^{x^2}$

$f'(x) = \underbrace{e^{x^2}(2x)}$

Ex.2 Find the equation of the tangent to the curve at x = -1 for

$$y = \frac{e^x}{x^2}$$

$m_{tan} = f'(-1)$

let $f(x) = \frac{e^x}{x^2}$

$f'(x) = \frac{e^x x^2 - e^x(2x)}{x^4}$

$f'(x) = \frac{x^2 e^x - 2x e^x}{x^4}$

$f'(-1) = \frac{(-1)^2 e^{-1} - 2(-1)e^{-1}}{(-1)^4}$

$= \frac{e^{-1} + 2e^{-1}}{1}$

$= 3e^{-1}$

$= \frac{3}{e}$ m_{tan}

just a number

$e \doteq 2.72$

$y = \overset{\checkmark}{m}x + \overset{?}{b}$

sub $P(x,y) = P(-1, ?)$

$f(-1) = \frac{e^{-1}}{(-1)^2}$

$= \frac{1}{e}$

Sub $P(-1, \frac{1}{e})$ into $y = \frac{3}{e}x + b$

$\frac{1}{e} = \frac{3}{e}(-1) + b$

$\frac{1}{e} = \frac{-3}{e} + b$

$\frac{1}{e} + \frac{3}{e} = b$

$b = \frac{4}{e}$

$\therefore$ eqn of tangent at $x = -1$ is $\boxed{y = \frac{3}{e}x + \frac{4}{e}}$

Assigned Work:

p. 232 # 1, 2aef, 3, 4bc,
5a, 7, 8, 12, 13

Deriv of e^x Demo.ggb