Recall: Given $f(x) = b^x$

$$f'(x) = b^x \lim_{h\to 0} \frac{b^h - 1}{h}$$

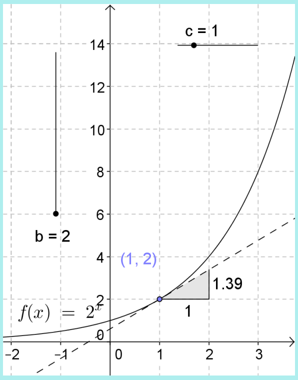
Notice that the limit does not depend on x, only b.
By taking the ratio,

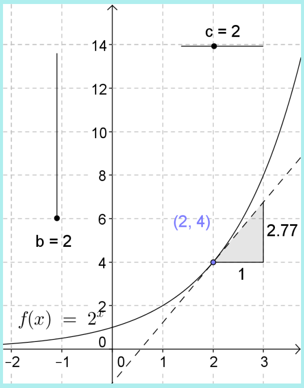
$$\frac{f'(x)}{f(x)} = \lim_{h\to 0} \frac{b^h - 1}{h}$$

we are able to isolate the limit. Now we consider this ratio from available data (graphing technology).

Let $f(x) = 2^x$

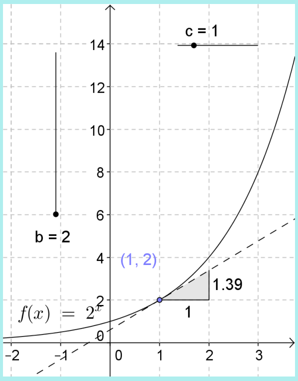
x	$f(x)$	$f'(x)$	$\frac{f'(x)}{f(x)}$
0	1	0.69	0.69
1			
2			
3			

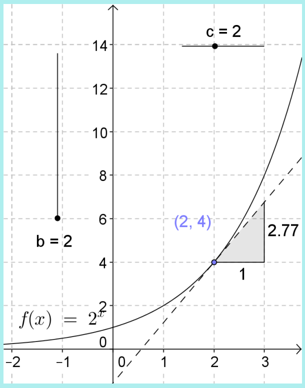




Let $f(x) = 2^x$

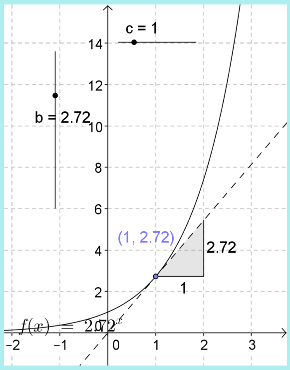
x	$f(x)$	$f'(x)$	$\frac{f'(x)}{f(x)}$
0	1	0.69	0.69
1	2	1.39	0.695
2	4	2.77	0.6925
3	8	5.55	0.69375

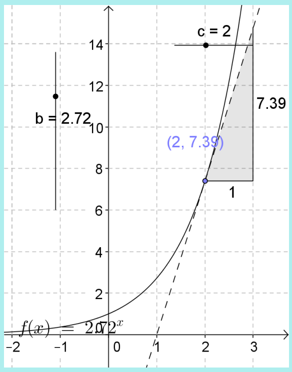




Let $f(x) = e^x$

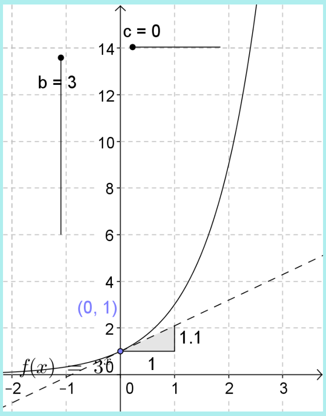
x	$f(x)$	$f'(x)$	$\frac{f'(x)}{f(x)}$
0	1	1	1
1	e	2.72	1
2	e^2	7.39	1
3	e^3	20.09	1

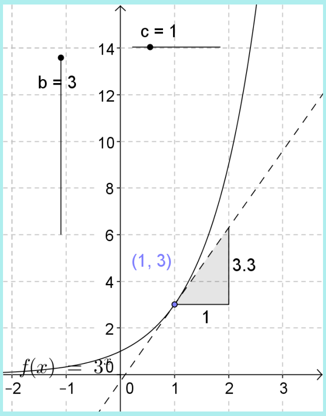




Let $f(x) = 3^x$

x	$f(x)$	$f'(x)$	$\frac{f'(x)}{f(x)}$
0	1	1.1	1.1
1	3	3.3	1.1
2	9	9.89	1.1
3	27	29.66	1.1





Can we represent this ratio with a function?

Is there a function g(x), such that:

$g(2) \doteq 0.69$ $\quad \log_2(2^x) = x$

$g(e^1) = 1$ $\quad \log_e(e^x)$

$g(3) \doteq 1.10$ $\quad \log_3(3^x)$

Focus on the result with 'e'. What function, with an input of 'e', would yield 1?

Consider the question another way: What function, with an input of 1, would yield 'e'?

$f(x)$

$g(f(x)) = x$?

$$g(\mathrm{e}) = 1$$

Focus on the result with 'e'. What function, with an input of 'e', would yield 1?

Consider the question another way: What function, with an input of 1, would yield 'e'?

$$f(1) = e$$

Derivative of General Exponential Functions

Apr. 5/2018

Consider $f(x) = e^x$, where 'e' is Euler's number, $e \doteq 2.718$.

From the graph, it is clearly a function, which means it must have an inverse (which turns out to also be a function).

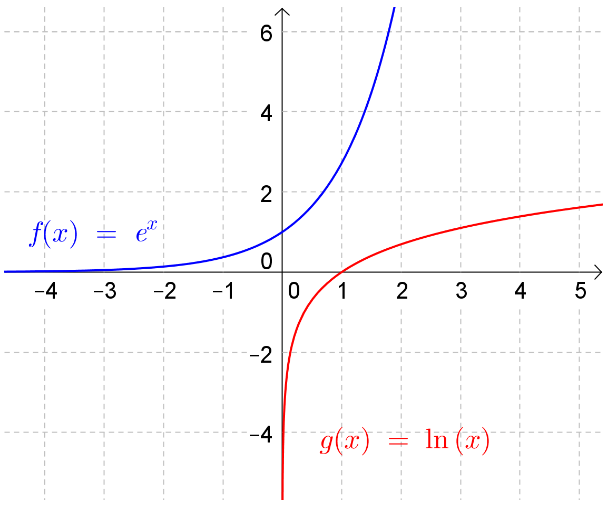
The inverse is

$$g(x) = \ln(x)$$

(pronounced "lawn") which is the natural logarithm.

It can also be written

$$g(x) = \log_e(x)$$



Given $f(x) = b^x$, we used limits to determine

$$f'(x) = b^x \lim_{h \to 0} \frac{b^h - 1}{h}$$

By using graphs and tangent lines, we can show (numerically) that:

$$\lim_{h \to 0} \frac{b^h - 1}{h} = \log_e(b) = \ln(b)$$

Therefore, given

$$f(x) = b^x \qquad f'(x) = \ln(b) b^x$$

$$f(x) = b^{g(x)} \qquad f'(x) = \ln(b) b^{g(x)} g'(x)$$

Ex. Determine the derivative of each.

(a) $f(x) = 7^x$ (b) $y = 5^{\sqrt{x}}$ (c) $g(x) = \frac{e^{x^2}}{3^x}$

(a) $f'(x) = \ln(7) \cdot 7^x$

or

$= 7^x \ln 7$

$\sqrt{x} = x^{\frac{1}{2}}$

$\frac{d}{dx}(x^{\frac{1}{2}}) = \frac{1}{2} x^{-\frac{1}{2}}$

(b) $y' = 5^{\sqrt{x}} \cdot \ln 5 \cdot \frac{1}{2} x^{-\frac{1}{2}}$

$= \frac{5^{\sqrt{x}} \cdot \ln 5}{2\sqrt{x}}$

(c) $g(x) = \frac{e^{x^2}}{3^x}$

$\frac{d}{dx}(e^{x^2})$

$= e^{x^2} \cdot (2x)$

$g'(x) = \frac{2x e^{x^2} \cdot 3^x - e^{x^2} \cdot 3^x \ln 3}{(3^x)^2}$

optional

$= \frac{e^{x^2} \cdot 3^x [2x - \ln 3]}{3^{2x}}$

$\frac{3^x}{3^{2x}} = 3^{x-2x}$

$= 3^{-x}$

$= \frac{1}{3^x}$

$= \frac{e^{x^2}[2x - \ln 3]}{3^x}$?

Assigned Work:

p.240 # 1, 2, 4, 5, 6, 9

Attachments

Deriv of e^x Demo.ggb