Exponential & Logarithmic Optimization

Apr. 11/2018

Strategy for solving optimization problems:

1. Draw a diagram (if possible).
2. Identify an equation in a single variable.
3. Domain of function.
4. Identify absolute max or min from extrema, end points.
5. Answer the problem.

For exponential functions, also consider:

(a) Rules for changing exponential expressions.
(b) Take log of both sides to isolate exponents.
(c) Limit behaviour & asymptotes.

Ex.1 Use the curve sketching algorithm to sketch

$$y = x^2 e^{2x}$$

(1) 1st and 2nd derivatives

$$y' = 2xe^{2x} + x^2e^{2x}(2)$$
$$= 2xe^{2x} + 2x^2e^{2x}$$
$$= 2xe^{2x}(1+x)$$
$$y'' = \left[2e^{2x} + 2xe^{2x}(2)\right] + \left[4xe^{2x} + 2x^2e^{2x}(2)\right]$$
$$= 2e^{2x} + 4xe^{2x} + 4xe^{2x} + 4x^2e^{2x}$$
$$= 2e^{2x} + 8xe^{2x} + 4x^2e^{2x}$$
$$= 2e^{2x}(1+4x+2x^2)$$

Ex.1 Use the curve sketching algorithm to sketch

(2) critical values

(+ PoI)

$$y = x^2 e^{2x}$$
$$y' = 2x(1+x)e^{2x}$$
$$y'' = 2(2x^2+4x+1)e^{2x}$$

$y' = 0$

$$2x(1+x)e^{2x} = 0$$

$$\underbrace{x = 0 \quad x = -1}_{\text{C.V.}} \qquad \text{no sol}$$

$y'' = 0$

$$0 = 2(2x^2+4x+1)e^{2x}$$

e^{2x}: no sol

$$x = \frac{-4 \pm \sqrt{16-8}}{4}$$
$$= \frac{-4 \pm \sqrt{8}}{4}$$
$$= \frac{-4 \pm 2\sqrt{2}}{4}$$
$$= \frac{-2 \pm \sqrt{2}}{2}$$
$$x = \frac{-2+\sqrt{2}}{2} \quad \text{OR} \quad x = \frac{-2-\sqrt{2}}{2}$$

Ex.1 Use the curve sketching algorithm to sketch

(3) 2nd derivative test

$$y = x^2 e^{2x}$$
$$0 = 2x(1+x)e^{2x} \rightarrow x = 0, x = -1$$
$$0 = 2(2x^2+4x+1)e^{2x} \rightarrow x \approx -0.293, x \approx -1.707$$

		-1.707	-0.293	
2	+	+	+	
e^{2x}	+	+	+	
$(2x^2+4x+1)$	+	−	+	
	CU	CD	CU	

$x=-1$ is a max.

$x=0$ is a min

Ex.1 Use the curve sketching algorithm to sketch

$$y = x^2 e^{2x}$$

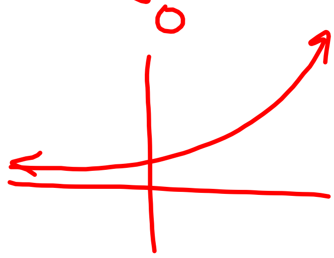
(4) end behaviour

$$\lim_{x \to \infty} \underbrace{x^2}_{+\infty} \underbrace{e^{2x}}_{+\infty}$$

$$= +\infty$$

$$\lim_{x \to -\infty} \underbrace{x^2}_{+\infty} \underbrace{e^{2x}}_{0}$$

$$= 0^+$$



Ex.1 Use the curve sketching algorithm to sketch

$$y = x^2 e^{2x}$$

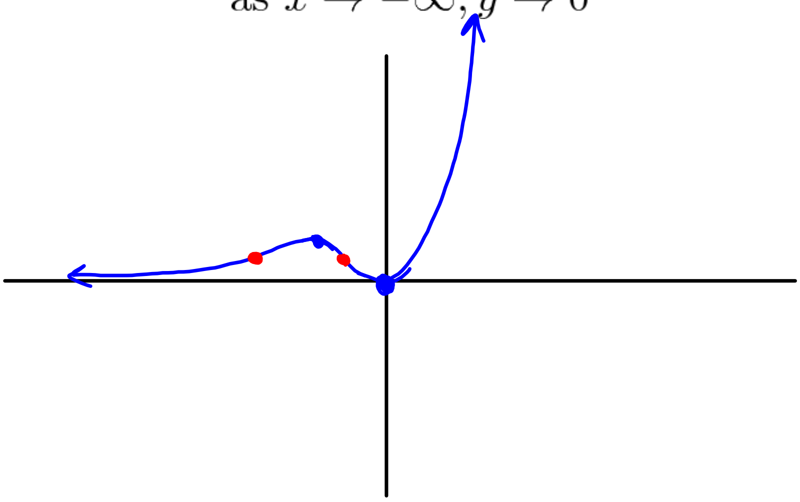
(5) sketch graph

$$\text{min} : (0, 0) \quad \text{max} : \left(-1, \frac{1}{e^2}\right) \approx (-1, 0.135)$$

$$\text{POI} : (-0.3, 0.05), (-1.7, 0.095)$$

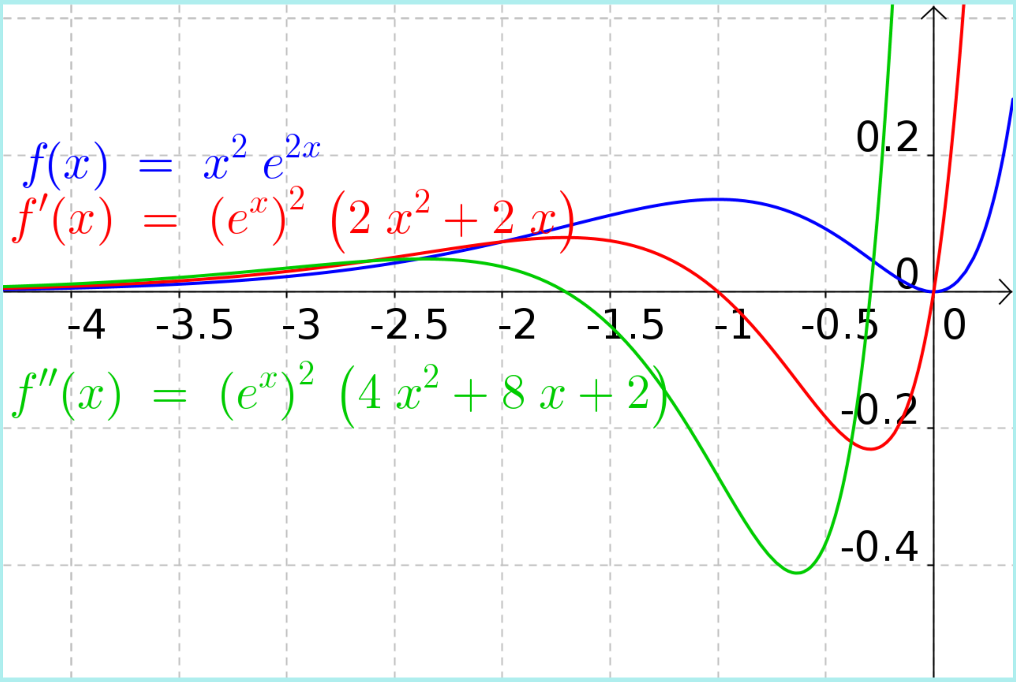
end behaviour: as $x \to \infty, y \to \infty$

as $x \to -\infty, y \to 0^+$



Ex.1 Use the curve sketching algorithm to graph

$$y = x^2 e^{2x}$$



Assigned Work:

Read examples 1 & 2, p.241-244

p. 245 # 3, 4, 5, 6, 8, 9, 12bc

c b

3(c) $P(t) = 20\left(1 + 3e^{-0.02t}\right)^{-1}$

$$P'(t) = 20(-1)\left(1+3e^{-0.02t}\right)^{-2}\left(3e^{-0.02t}\right)$$

$$= 0.4\left(\quad\right)^{-2}\left(3e^{-0.02t}\right) \times(-0.02)$$

$$= 1.2e^{-0.02t}\left(1+3e^{-0.02t}\right)^{-2}$$

$$= \frac{1.2e^{-0.02t}}{\left(1+3e^{-0.02t}\right)^{2}}$$

$$P''(t) = \frac{1.2e^{-0.02t}(-0.02)\left(1+3e^{-0.02t}\right)^{2} - 1.2e^{-0.02t}(2)\left(1+3e^{-0.02t}\right)^{1}\left(3e^{-0.02t}(-0.02)\right)}{\left(1+3e^{-0.02t}\right)^{4}}$$

Set $P''(t) = 0$

$$0 = -0.02\left(1.2e^{-0.02t}\right)\left(1+3e^{-0.02t}\right)\left[1+3e^{-0.02t} - (2)\left(3e^{-0.02t}\right)\right]$$

$$0 = \underbrace{-\underbrace{0.024e^{-0.02t}}_{>0}}_{<0}\underbrace{\left(1+3e^{-0.02t}\right)}_{>0}\underbrace{\left(1+3e^{-0.02t}-6e^{-0.02t}\right)}$$

$$1 - 3e^{-0.02t} = 0$$

$$1 = 3e^{-0.02t}$$

$$\frac{1}{3} = e^{-0.02t}$$

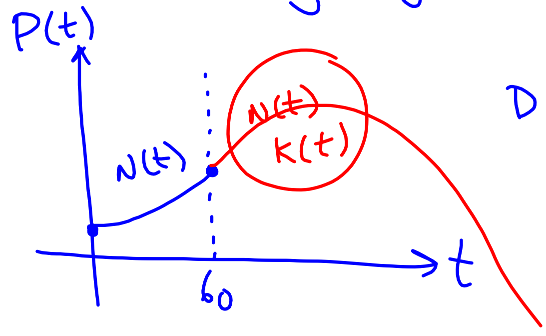
$$\ln\left(\frac{1}{3}\right) = \ln\left(e^{-0.02t}\right)$$

$$\ln\frac{1}{3} = -0.02t$$

$$t = \frac{\ln\left(\frac{1}{3}\right)}{-0.02}$$

8(b) $N(t) = 2^{\frac{t}{5}}$ add $K(t) = e^{\frac{t}{3}}$ Subtract

* 60 minute delay before killing drug introduced



$D: t > 60$

$$P(t) = N(t) - K(t-60)$$

start of K(t) is shifted right by 60

$$P(t) = 2^{\frac{t}{5}} - e^{\frac{t-60}{3}}$$

9. $E_1 = 0.6\left(9 + te^{-\frac{t}{20}}\right)$

$E_2 = 0.5\left(10 + te^{-\frac{t}{10}}\right)$

$t_T = 30$

$x + y = 30$

$y = 30 - x$

$f(t) = E_1(t) + E_2(t)$

Let x represent the time studying for Test 1

$f(x) = E_1(x) + E_2(30 - x)$