Derivative of the Tangent Function

Apr. 13/2018

Ex.1 Evaluate $\frac{d}{dx}tan(x)$

$$= \frac{d}{dx}\left(\frac{\sin x}{\cos x}\right)$$

$$= \frac{\cos x \cdot \cos x - \sin x(-\sin x)}{\cos^2 x}$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$$

$$= \frac{1}{\cos^2 x}$$

$$= \sec^2 x$$

$$\frac{d}{dx}(tanx) = sec^2 x$$

Chain Rule:

Given $y = tan\left[g(x)\right]$

$$\frac{dy}{dx} = sec^2\left[g(x)\right]g'(x)$$

Ex. Determine the first derivative for each.

(a) $f(x) = \tan(4x)$ (b) $f(x) = \sin(x)\tan(x)$

(c) $y = \tan^2(\sqrt{x})$ (d) $y = \ln(\tan(x^3))$

(a) $f'(x) = 4\sec^2(4x)$

(b) $f'(x) = \cos x \tan x + \sin x \sec^2 x$

$= \cos x \cdot \frac{\sin x}{\cos x} + \sin x \cdot \frac{1}{\cos^2 x}$

$= \sin x + \tan x \sec x$

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(c) $y' = (2\tan^{1}\sqrt{x})(\sec^2\sqrt{x})(\frac{1}{2}x^{-\frac{1}{2}})$

$= \frac{\tan\sqrt{x}\,\sec^2\sqrt{x}}{\sqrt{x}}$

(d) $y' = \frac{1}{\tan(x^3)} \cdot \sec^2(x^3) \cdot (3x^2)$

$= \frac{3x^2\sec^2(x^3)}{\tan(x^3)}$ ✓

$= 3x^2 \cdot \frac{1}{\cos^2(x^3)} \cdot \frac{\cos(x^3)}{\sin(x^3)}$

$= \frac{3x^2}{\sin(x^3)\cos(x^3)}$

$= 3x^2\csc(x^3)\sec(x^3)$ $= \frac{3x^2}{\frac{1}{2}\sin(2x^3)}$

Assigned Work:

p.259 # 1-8, [10, 11]

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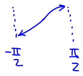
7. $y = \sec x + \tan x$ ↑ $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$



- positive slope, $y' > 0$

① look for C.V. $(y' = 0)$

② classify int. table

$y = (\cos x)^{-1} + \tan x$

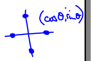
$y' = -(\cos x)^{-2}(-\sin x) + \sec^2 x$

$y' = \frac{\sin x}{\cos^2 x} + \frac{1}{\cos^2 x}$

for C.V., set $y' = 0$

$0 = \frac{1}{\cos^2 x}\left[\sin x + 1\right]$

$\frac{1}{\cos^2 x} = 0$ or $\sin x + 1 = 0$

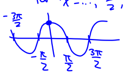


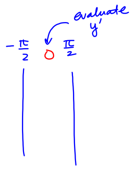
$1 = 0$ — no solution

$\sin x = -1$

$x = \ldots, -\frac{\pi}{2}, \frac{3\pi}{2}, \ldots$

y' is undefined for $x = \ldots, -\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, \ldots$





y' at $x = 0$

$y' = \frac{\sin 0}{\cos^2 0} + \frac{1}{\cos^2 0}$

$= \frac{0}{1} + \frac{1}{1}$

$= 0 + 1$

$= 1$

> 0

from CV/interval table, y' is the same for all values between $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$y' > 0$ (positive slope)

∴ y is always increasing on $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$