Scalar Multiplication of a Vector

Starting with any vector $\vec{a}$, then $k\vec{a}$ represents a scalar multiplication by a factor of k.

k can also be referred to as the scale factor, and it can have the following effects:

$|k| > 1$, stretch

$|k| < 1$, compress

$k < 0$, reflect (i.e., opposite direction)

Effects of scalar multiplication, $k\vec{a}$, on vector $\vec{a}$

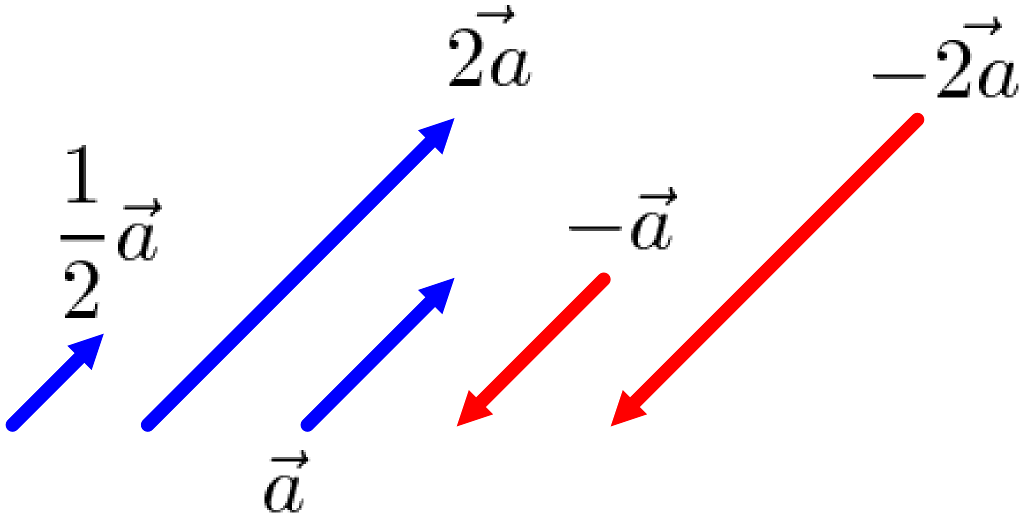
k	effect	illustration
$k > 1$	stretch, same direction	
$k = 1$	original vector	
$0 < k < 1$	compress, same direction	
$k = 0$	zero vector $\vec{0}$	
$-1 < k < 0$	compress, opposite direction	
$k = -1$	opposite vector	
$k < -1$	stretch, opposite direction	

Collinear vectors are parallel (or can be arranged to lie on the same straight line). All collinear vectors are scalar multiples of each other.

Two vectors are said to be collinear if and only if:

$$\vec{a} = k\vec{b}, \text{ where } k \in \mathbb{R}$$

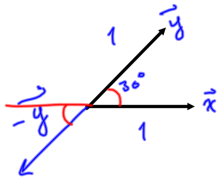
Some examples:

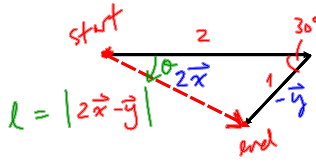


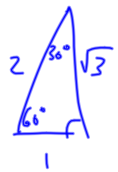
As previously discussed, a vector has both magnitude and direction. There are many ways to express the direction of a vector depending upon the situation.

Ex.1 Vectors x and y are unit vectors (→ magnitude = 1) that make an angle of 30 degrees between them. Determine $2\vec{x} - \vec{y}$

$= 2\vec{x} + (-\vec{y})$







$a^2 = b^2 + c^2 - 2bc\cos A$

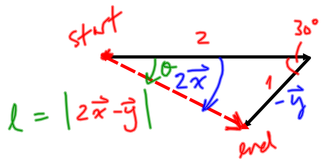
$|2\vec{x} - \vec{y}|^2 = 2^2 + 1^2 - 2(2)(1)\cos 30°$

$|2\vec{x} - \vec{y}|^2 = 4 + 1 - 4\left(\frac{\sqrt{3}}{2}\right)$

$|2\vec{x} - \vec{y}|^2 = 5 - 2\sqrt{3}$

$|2\vec{x} - \vec{y}| = \sqrt{5 - 2\sqrt{3}}$, $l > 0$ ✓

$\doteq 1.24$



$\frac{\sin\theta}{1} = \frac{\sin 30°}{|2\vec{x} - \vec{y}|}$

$\sin\theta = \frac{\frac{1}{2}}{\sqrt{5 - 2\sqrt{3}}}$

Q2 incl $(>180°)$

RAA $\doteq 23.8°$

$\theta = 23.8°$

$\therefore 2\vec{x} - \vec{y} \doteq 1.24$ [23.8° CW to $\vec{x}$]

Ex.2 An airplane flies N30° E at an airspeed of 240 km/h.

(a) Represent velocity using vector notation.

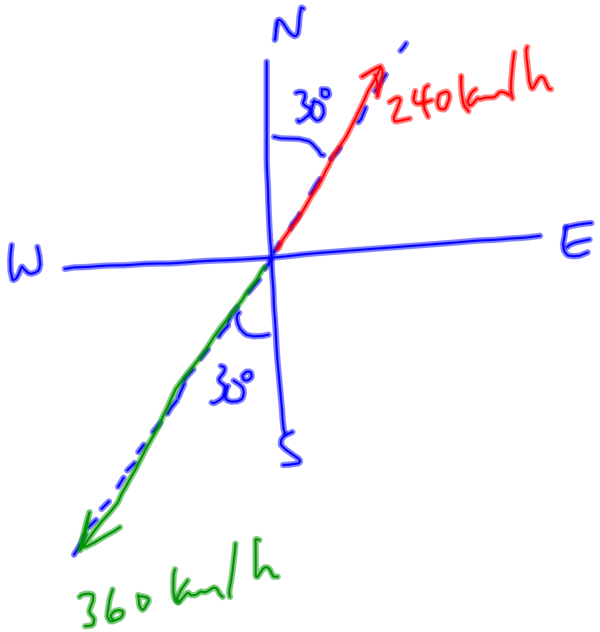
(b) Sketch this situation.

(c) The return flight occurs at 150% of the original speed. Sketch and represent using vector notation.

(a) $\vec{v_1} = 240 \text{ km/h } [N\,30^\circ E]$

$= 240 \text{ km/h } [30^\circ \text{ E of N}]$

(b)



(c) $k = -1.5$

$\vec{v_2} = -1.5\,\vec{v_1}$

$= -1.5(240)\,[N\,30^\circ E]$

$= -360\,[N\,30^\circ E]$

$= 360\,[S\,30^\circ W]$

In math, physics, and other fields making use of vectors, we are often interested in a unit vector.

A unit vector is collinear to a given vector, having a magnitude (i.e., length) of one.

Given vector $\vec{v}$, the corresponding unit vector is $\vec{u} = \dfrac{\vec{v}}{|\vec{v}|}$

$$\vec{v} = 5\,[N]$$

$$\vec{u} = \frac{\vec{v}}{|\vec{v}|}$$

$$= \frac{\cancel{5}^{1}\,[N]}{\cancel{5}_{1}}$$

$$\vec{u} = 1\,[N]$$

Assigned Work:

p.299 # 1, 5, 7, 12, 14, 15, 19, 21*