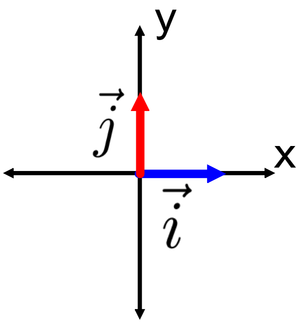
Operations with Vectors in R^2

Apr. 26/2018

Recall: A unit vector is a vector of length one (1).



For unit vectors along the x- and y-axes, we use:

$$\vec{i} = (1,0) \qquad \hat{x}$$
$$\vec{j} = (0,1) \qquad \hat{y}$$

These are the standard basis vectors for R^2, meaning that any vector in R^2 can be expressed in terms of **i** and **j**.

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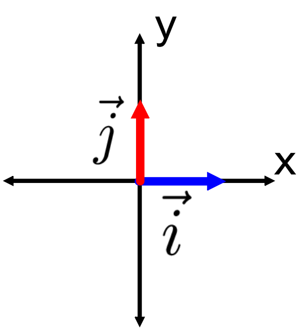

$$\vec{i} = (1,0)$$
$$\vec{j} = (0,1)$$

Any point, P(a,b), can be represented as an algebraic vector expressed in terms of the unit vectors:

$$\overrightarrow{OP} = (a,b) = a\vec{i} + b\vec{j}$$

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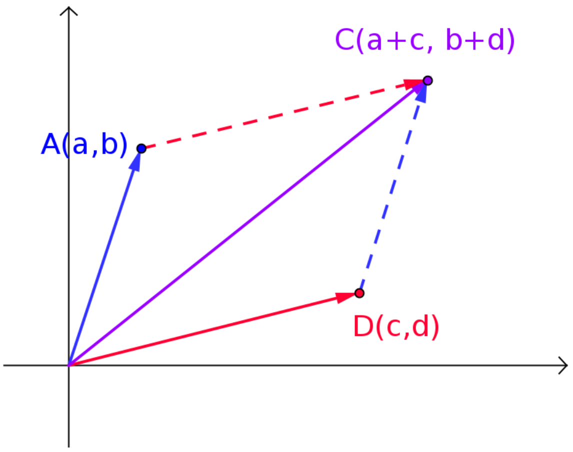
Adding Vectors



$$\vec{OC} = \vec{OA} + \vec{OD}$$
$$= (a\vec{i} + b\vec{j}) + (c\vec{i} + d\vec{j})$$
$$= (a+c)\vec{i} + (b+d)\vec{j}$$
$$= (a+c,\ b+d)$$

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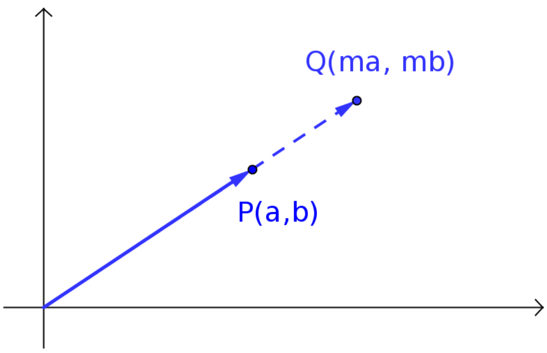
Scalar Multiplication of Vectors

Given $P(a, b)$



$$\vec{OQ} = m\,\vec{OP}$$
$$= m(a, b)$$
$$= m(a\vec{i} + b\vec{j})$$
$$= ma\vec{i} + mb\vec{j}$$
$$= (ma, mb)$$

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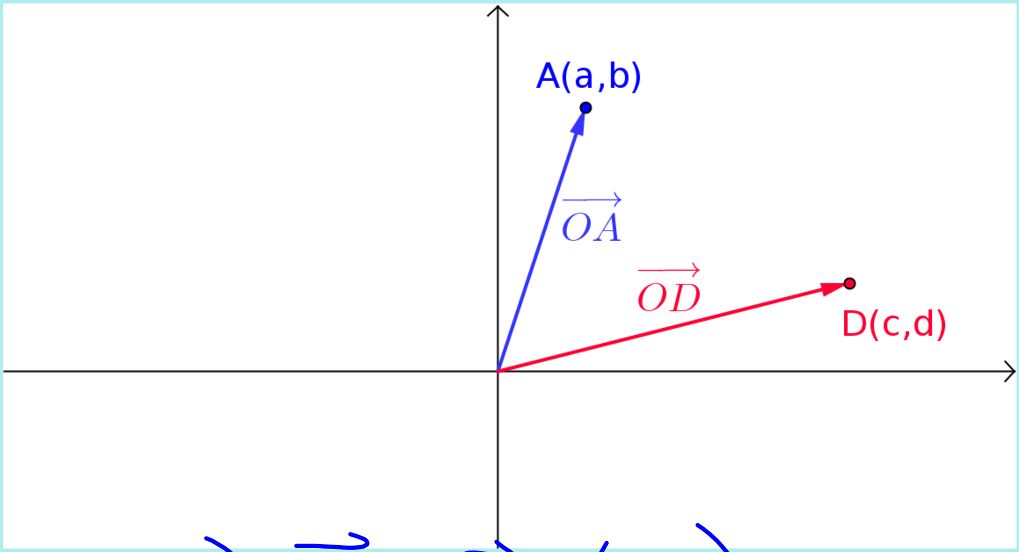
Difference of Vectors



$\overrightarrow{OA} - \overrightarrow{OD} = \overrightarrow{OA} + \left(-\overrightarrow{OD}\right)$

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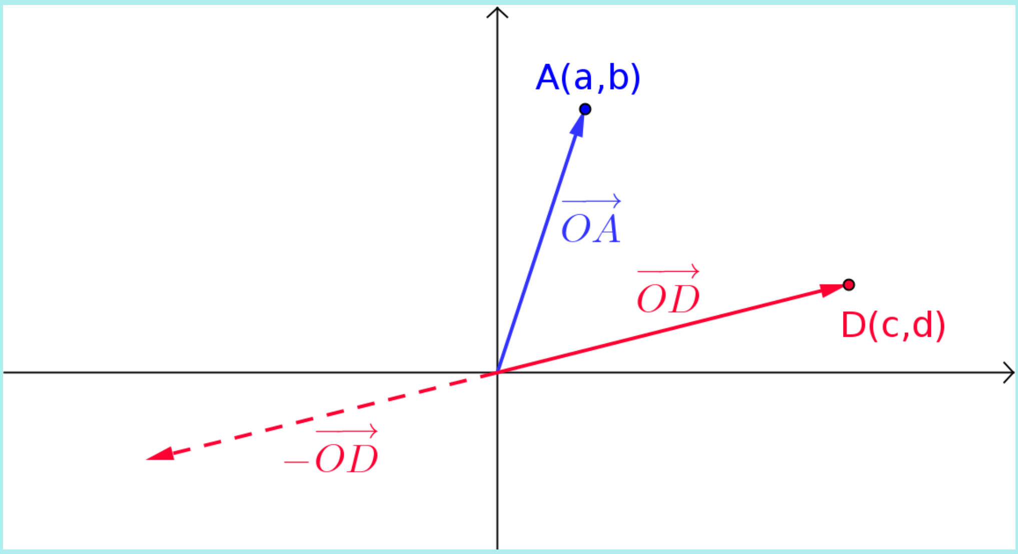
Difference of Vectors



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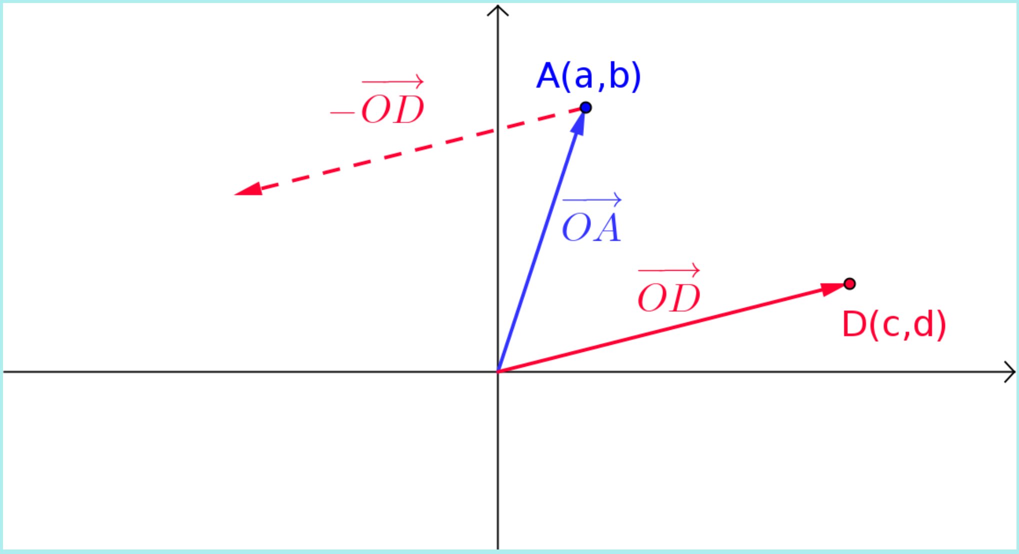
Difference of Vectors



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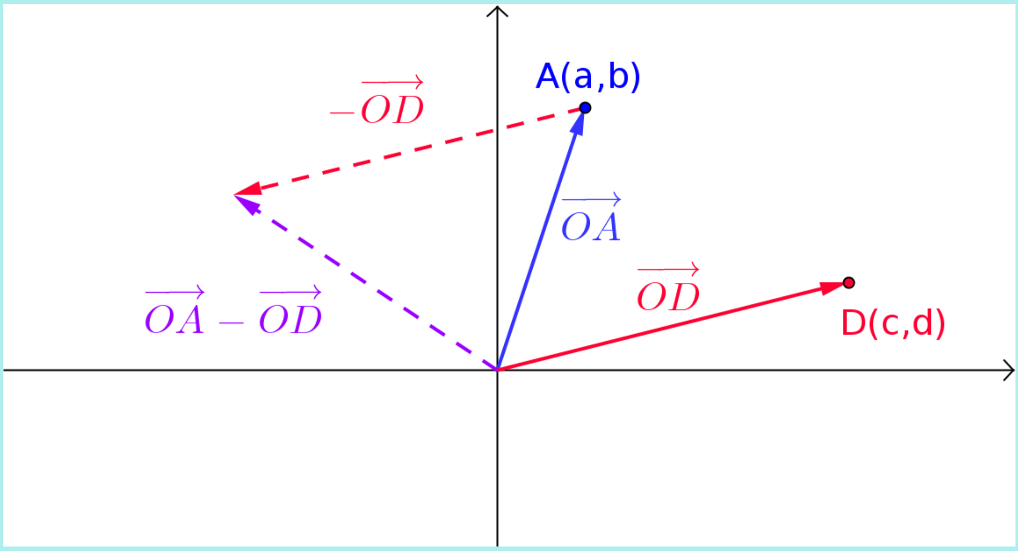
Difference of Vectors



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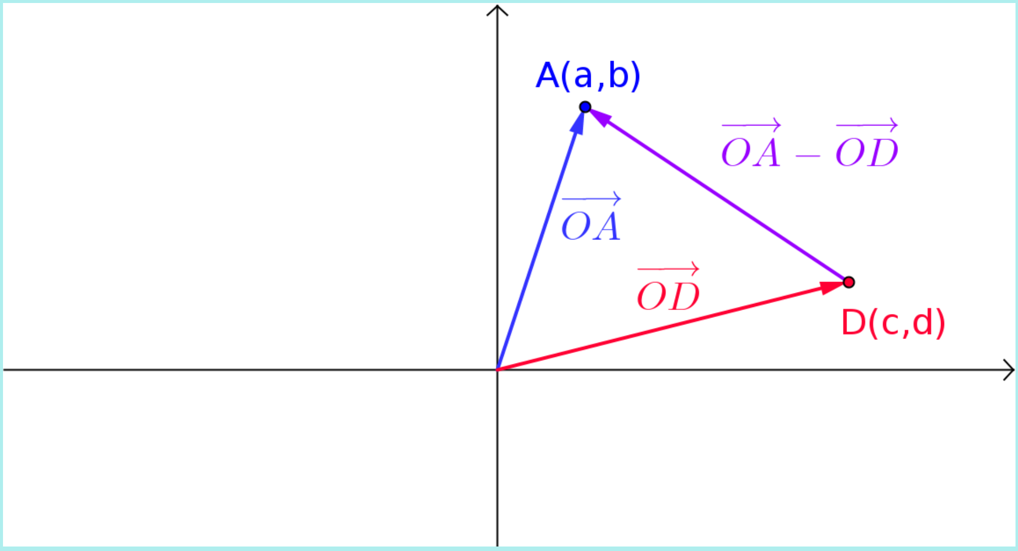
Difference of Vectors



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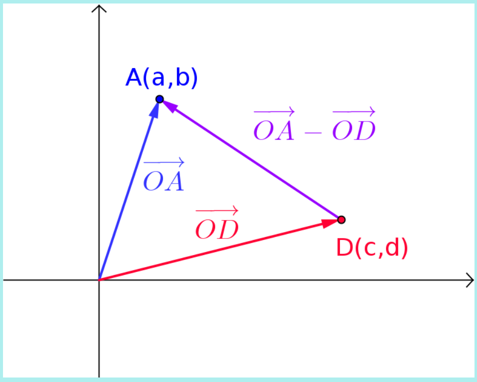
Difference of Vectors

The difference of two position vectors gives the vector between the original points (from the second to the first).



$$
\begin{aligned}
\overrightarrow{OA} - \overrightarrow{OD} &= (a,b) - (c,d) \\
&= a\vec{i} + b\vec{j} - (c\vec{i} + d\vec{j}) \\
&= a\vec{i} + b\vec{j} - c\vec{i} - d\vec{j} \\
&= (a-c)\vec{i} + (b-d)\vec{j} \\
&= (a-c, b-d) \\
&= \overrightarrow{DA}
\end{aligned}
$$

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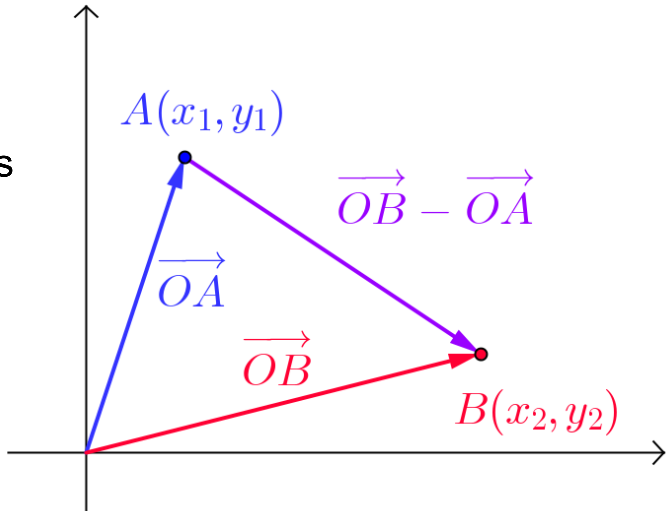

$$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$$

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Difference of Vectors

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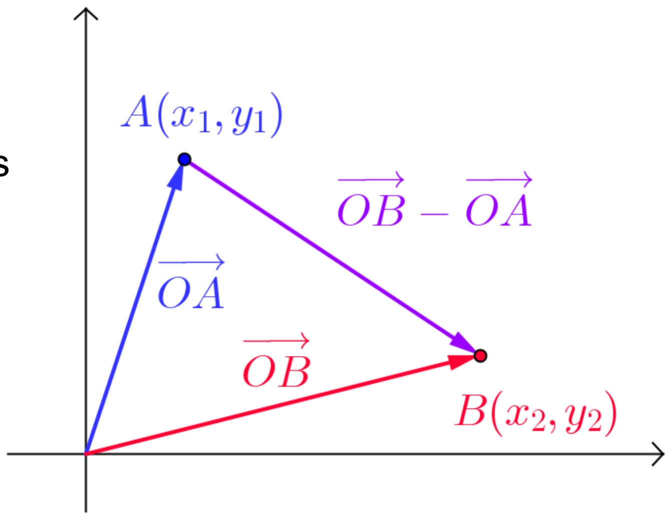

$$\left|\overrightarrow{AB}\right| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

which is the distance formula between points A and B.

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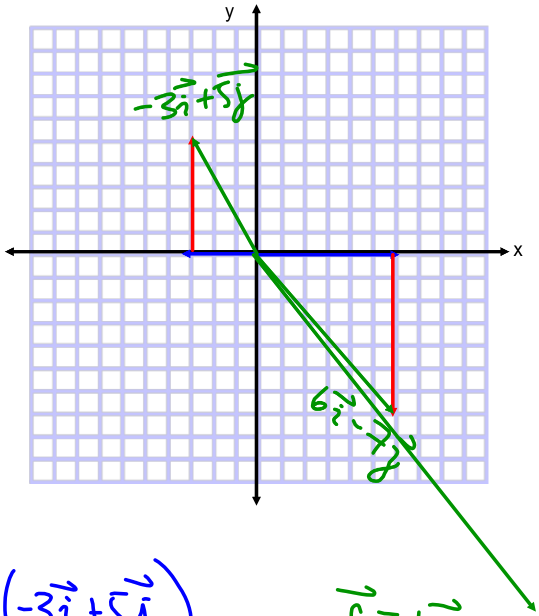
Ex.1 Draw the following vectors in two dimensions.

$\vec{a} = 6\vec{i} - 7\vec{j}$

$\vec{b} = -3\vec{i} + 5\vec{j}$

$\vec{c} = \vec{a} - 2\vec{b}$



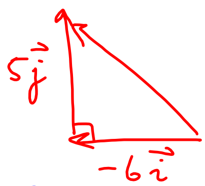
$$\begin{aligned}\vec{c} &= \vec{a} - 2\vec{b}\\ &= (6\vec{i}-7\vec{j}) - 2(-3\vec{i}+5\vec{j})\\ &= 6\vec{i}-7\vec{j}+6\vec{i}-10\vec{j}\\ &= 12\vec{i}-17\vec{j}\end{aligned}$$

$\vec{c} = 12\vec{i} - 17\vec{j}$

May 2-8:43 AM

Ex.2 Given the points L(4,-2) and M(-2,3), find:
(a) $\overrightarrow{LM}$ and $|\overrightarrow{LM}|$
(b) a unit vector in the direction of $\overrightarrow{LM}$

(a) $$\begin{aligned}\overrightarrow{LM} &= \overrightarrow{OM} - \overrightarrow{OL}\\ &= (-2,3) - (4,-2)\\ &= (-2\vec{i}+3\vec{j}) - (4\vec{i}-2\vec{j})\\ &= -6\vec{i}+5\vec{j}\\ &= (-6,5)\end{aligned}$$



$$\begin{aligned}|\overrightarrow{LM}| &= d_{LM}\\ &= \sqrt{(-6)^2+(5)^2}\\ &= \sqrt{61}\end{aligned}$$

(b) $$\begin{aligned}\widehat{LM} &= \frac{\overrightarrow{LM}}{\sqrt{61}}\\ &= \frac{1}{\sqrt{61}}(-6,5)\end{aligned}$$

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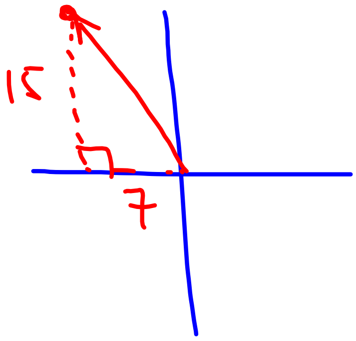
Ex.3 Given $\vec{a} = \vec{i} - 5\vec{j}$ and $\vec{b} = 4\vec{i} - 10\vec{j}$, find $\left|\vec{a} - 2\vec{b}\right|$.

NOTE: Solving problems using algebraic vectors (distance formula) can sometimes be simpler than solving them using geometric vectors (cosine law).

$$\vec{a} - 2\vec{b} = (\vec{i} - 5\vec{j}) - 2(4\vec{i} - 10\vec{j})$$
$$= \vec{i} - 5\vec{j} - 8\vec{i} + 20\vec{j}$$
$$= -7\vec{i} + 15\vec{j}$$



$$|\vec{a} - 2\vec{b}| = \sqrt{(-7)^2 + (15)^2}$$
$$= \sqrt{49 + 225}$$
$$= \sqrt{274}$$

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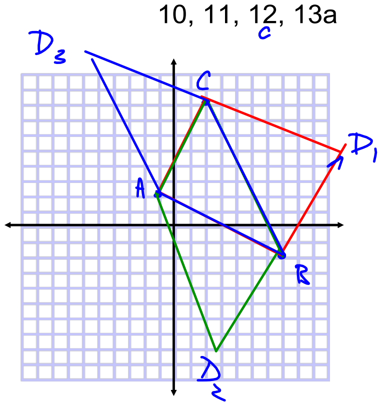
Assigned Work:

p.325 # 3, 4, 5b, 6ac, 7c, 8c, 10, 11, 12c, 13a



$\vec{BD_1} = \vec{AC}$

$\vec{OD_1} = \vec{OB} + \vec{BD_1}$

$= (7,-2) + \vec{AC}$ $\quad \vec{AC} = \vec{OC} - \vec{OA}$

$= (7,-2) + \vec{OC} - \vec{OA}$

$= (7,-2) + (2,8) - (-1,2)$

$= 7\vec{i} - 2\vec{j} + 2\vec{i} + 8\vec{j} + \vec{i} - 2\vec{j}$

$= 10\vec{i} + 4\vec{j}$

$\vec{OD_1} = (10,4)$ $\quad \therefore D_1$ is $P(10,4)$

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