Algebraic Vectors in R^3

Apr.30/2018

The unit vectors $\vec{i} = (1,0,0)$

$$\vec{j} = (0,1,0)$$

$$\vec{k} = (0,0,1)$$

are the standard basis vectors in R^3.

Given the point P(a,b,c), the position vector is

$$\begin{aligned}\overrightarrow{OP} &= (a,b,c)\\ &= a\vec{i} + b\vec{j} + c\vec{k}\end{aligned}$$

$$\left|\overrightarrow{OP}\right| = \sqrt{a^2 + b^2 + c^2}$$

Apr 29-6:54 AM

Given the points $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$

$$\begin{aligned}\overrightarrow{AB} &= \overrightarrow{OB} - \overrightarrow{OA}\\ &= (x_2, y_2, z_2) - (x_1, y_1, z_1)\\ &= (x_2 - x_1)\vec{i} + (y_2 - y_1)\vec{j} + (z_2 - z_1)\vec{k}\\ &= (x_2 - x_1, y_2 - y_1, z_2 - z_1)\end{aligned}$$

$$\left|\overrightarrow{AB}\right| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Apr 29-7:00 AM

Ex. If A(2,3,4) and B(-4,5,-6) are two points in R^3, determine each of the following:

(a) $|\overrightarrow{OA}|$ (b) $|\overrightarrow{OB}|$ (c) $\overrightarrow{AB}$ (d) $|\overrightarrow{AB}|$

(a) $\overrightarrow{OA} = (2,3,4)$

$|\overrightarrow{OA}| = \sqrt{2^2+3^2+4^2}$

$= \sqrt{4+9+16}$

$= \sqrt{29}$

(b) $\overrightarrow{OB} = (-4,5,-6)$ $|\overrightarrow{OB}| = \sqrt{(-4)^2+5^2+6^2}$

$= \sqrt{16+25+36}$

$= \sqrt{77}$

(c) $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

$= (-4,5,-6) - (2,3,4)$

$= (-6, 2, -10)$

(d) $|\overrightarrow{AB}| = \sqrt{(-6)^2+2^2+(-10)^2}$

$= \sqrt{140}$

$= 2\sqrt{35}$

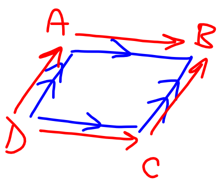
Apr 24-1:32 PM

Assigned Work:

p.332 # 1, 2, 3, 4, 5ac, 6bd, 8, 10, 11, 12, 14, 15

11. $A(0,3,5)$ $B(3,-1,17)$

$C(7,-3,15)$ $D(4,1,3)$



$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

$= (3,-1,17) - (0,3,5)$

$= (3,-4,12)$

$\overrightarrow{CD} = \overrightarrow{OD} - \overrightarrow{OC}$

$= (-3,4,-12)$

$= -\overrightarrow{AB}$

$\therefore \overrightarrow{AB} = -\overrightarrow{CD} = \overrightarrow{DC}$

$\therefore |\overrightarrow{AB}| = |\overrightarrow{CD}|$ and $\overrightarrow{AB} \parallel \overrightarrow{CD}$

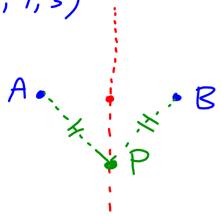
* do same for $\overrightarrow{BC}$ and $\overrightarrow{AD}$

May 4-9:19 AM

14. $A(-2,1,3)$ $B(4,-1,3)$

P on x-axis
$(x,0,0)$

on x-axis,
$y=0,\ z=0$



Want $d_{AP} = d_{PB}$
$|\vec{AP}| = |\vec{BP}|$

$\vec{AP} = \vec{OP} - \vec{OA}$
$= (x,0,0) - (-2,1,3)$
$= (x+2,-1,-3)$

$\vec{BP} = \vec{OP} - \vec{OB}$
$= (x-4,1,-3)$

$|\vec{AP}| = |\vec{BP}|$

$$\sqrt{(x+2)^2+(-1)^2+(-3)^2} = \sqrt{(x-4)^2+(1)^2+(-3)^2}$$

$$\cancel{x^2}+4x+4+\cancel{10} = \cancel{x^2}-8x+16+\cancel{10}$$

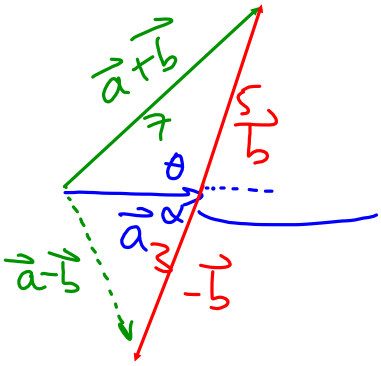
$$12x = 12$$

$$x = 1$$

May 1-12:42 PM

15. $|\vec{a}| = 3$ $|\vec{b}| = 5$ $|\vec{a}+\vec{b}| = 7$

$|\vec{a}-\vec{b}| = ?$



$\sqrt{9+25} < 6$

$\alpha = 180° - \theta$

① find θ using cosine law.
② find $|\vec{a}-\vec{b}|$ using α and cosine law.

May 1-12:53 PM