Linear Combinations & Spanning Sets

May 1/2018

Given noncollinear vectors **u** and **v**, a linear combination of these vectors is:

$$a\vec{u} + b\vec{v}$$

where a and b are scalars.

$\vec{i}$ $\vec{j}$

$\vec{w} = 4(1,0) + 23(0,1)$

Ex.1 Show that the vector, $\vec{w} = (4, 23)$ can be written as a linear combination of $\vec{u} = (-1, 4)$ and $\vec{v} = (2, 5)$

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$\vec{w} = a\vec{u} + b\vec{v}$, solve for a and b

$(4,23) = a(-1,4) + b(2,5)$

$(4,23) = (-a+2b, 4a+5b)$

x y x y

$(4 = -a+2b) \times 4$ $23 = 4a + 5b$

$16 = -4a + 8b \rightarrow 16 = -4a + 8b$

$39 = 13b$

$b = 3$

$4 = -a + 2(3)$

$a = 2$

$\therefore \vec{w} = 2\vec{u} + 3\vec{v}$

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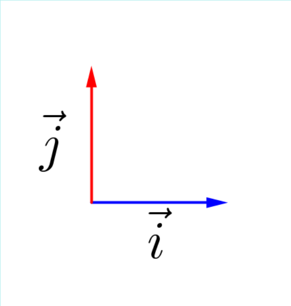
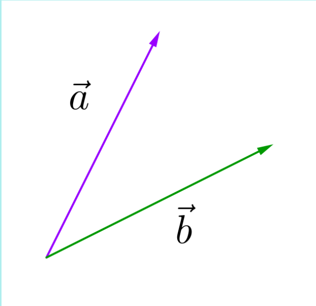
Consider two pairs of vectors:

$$\vec{i} = (1,0) \qquad \vec{a} = (1,2)$$
$$\vec{j} = (0,1) \qquad \vec{b} = (2,1)$$

Can either, or both, of these pairs be used to describe an arbitrary position vector in R^2?

For convenience, we will consider only $\vec{OP} = (6,6)$





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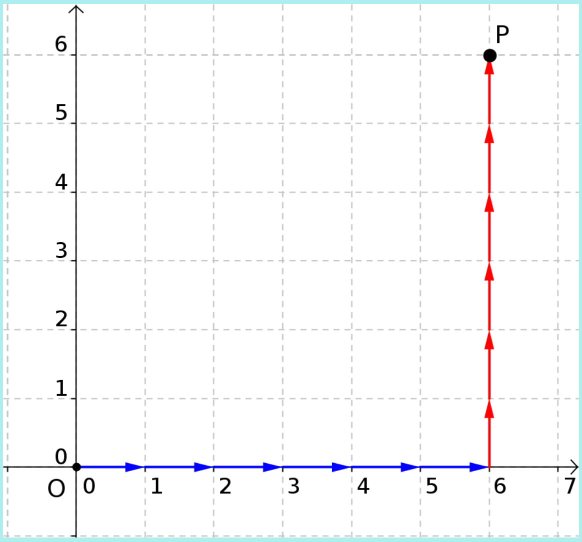
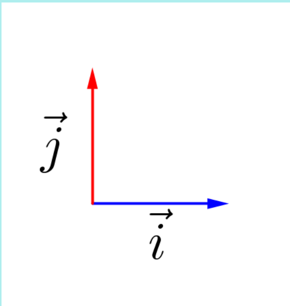
Clearly, vectors **i** and **j** are the obvious choice. They are the basis vectors in R^2, and can describe combination of x- and y- coordinates.





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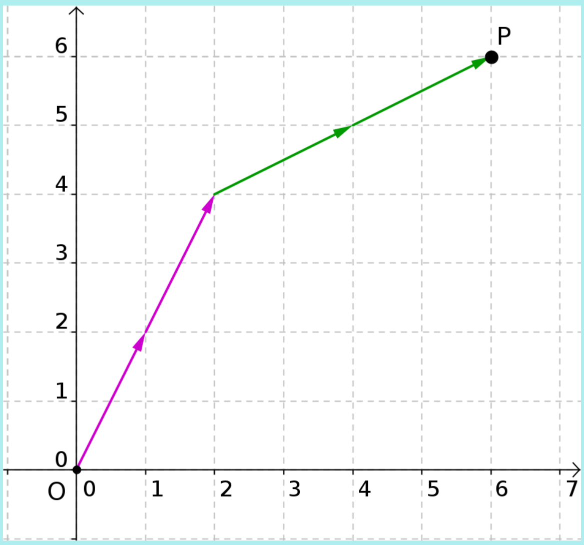
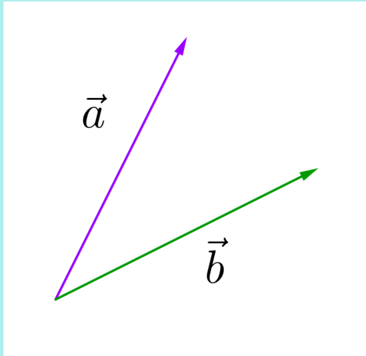
It turns out, however, that **a** and **b** are also valid. They are not unit vectors, they are not orthogonal, but they are non-collinear and non-zero, which is sufficient.





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Every vector in R^2 can be written uniquely as a linear combination of the unit vectors, $\vec{i}$ and $\vec{j}$.

This is actually true for any pair of nonzero, noncollinear vectors in the x-y plane. Such a pair of vectors is called a <u>spanning set</u> in R^2.

Ex.2 Prove that the vectors {(2,1), (-3,-1)} form a spanning set in R^2. ($\vec{a}$, $\vec{b}$)

non-zero : by inspection is okay.

$|\vec{a}| = \sqrt{2^2+1^2} = \sqrt{5} > 0$ $\quad$ $|\vec{b}| = \sqrt{(-3)^2+(-1)^2} = \sqrt{10} > 0$

non-collinear : test collinear, show contradiction

$\vec{a} = k\vec{b}$

$(2,1) = k(-3,-1)$

$(2,1) = (-3k,-k)$

$2 = -3k$ $\quad$ $1 = -k$

test $k=-1$ $\quad$ $k=-1$

$LS = 2$ $\quad$ $RS = -3(-1) = 3$

$LS \neq RS$

$\therefore \vec{a} \neq k\vec{b}$ $\quad$ $\therefore \vec{a}$ & $\vec{b}$ are <u>not</u> collinear.

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In R^3, any pair of nonzero, noncollinear vectors will span a plane (but not necessarily the x-y plane). Any vector in the same plane can be expressed as a linear combination of those vectors.

Corollary: Any vector which is a linear combination of those vectors must lie in the same plane as those vectors.

Ex.3 Show that the vector (-9,-4,1) [$\vec{w}$] lies in a plane defined by the vectors (-1,-2,1) [$\vec{u}$] and (3,-1,1) [$\vec{v}$].

Show $\vec{w} = a\vec{u} + b\vec{v}$ works with no contradictions or inconsistencies

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Ex.3 Show that the vector (-9,-4,1) lies in a plane defined by the vectors (-1,-2,1) and (3,-1,1).

$(-9,-4,1) = a(-1,-2,1) + b(3,-1,1)$

$-9 = -a + 3b$ ① $-4 = -2a - b$ ② $1 = a + b$ ③

③ $1 = a + b$

$-8 = 4b$

$b = -2$ → ③ $1 = a + (-2)$

$a = 3$

Verify using ② $-4 = -2a - b$

LS $= -4$ RS $= -2(3) - (-2)$

LS = RS $= -6 + 2$

$= -4$

∴ the vector lies in the plane.

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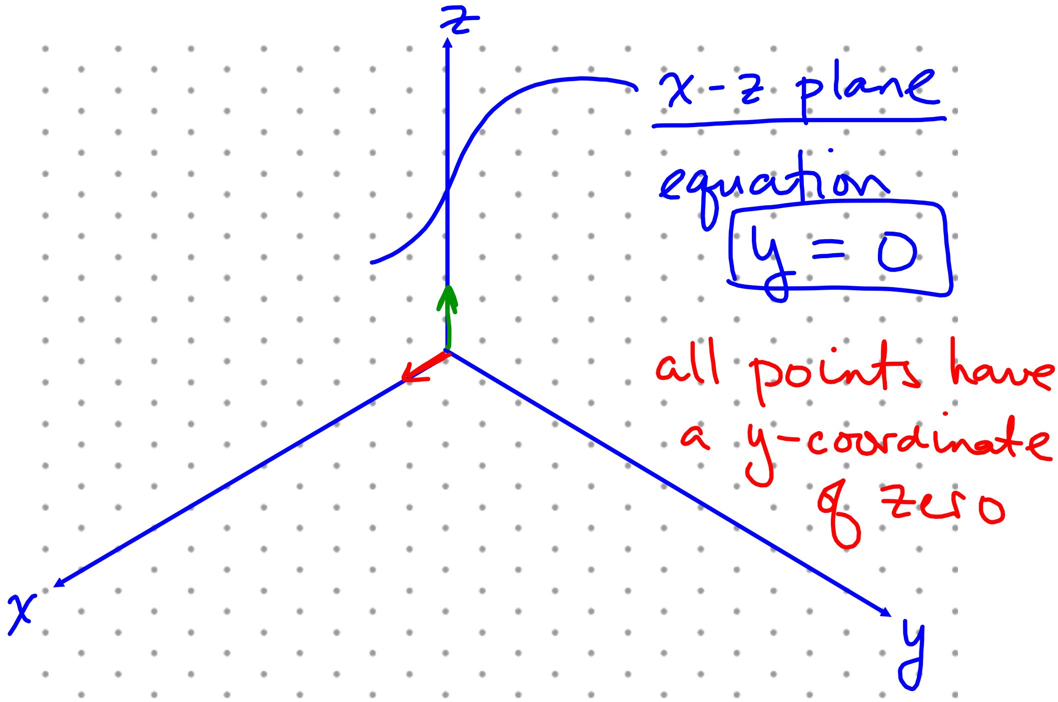
Ex.4 The vector set {(1, 0, 0), (0, 0, 1)} spans a plane in $\mathbf{R}^3$. What is the equation of the plane?



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Assigned Work:

p.340 # 6, 7b, 8, 10, 11, 13, 14

6. $\{(-1,2), (2,-4), (-1,1), (-3,6), (1,0)\}$

$\times(-2)$, $\times 3$

$\{(-1,2), (-1,1), (1,0)\}$
① ② ③

{①, ②} or {①, ③} or {②, ③}

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8. x-y plane has the eq. $z=0$.

Set #1: $\{(1,0,0),(0,1,0)\}$

$z=0$

Set #2: $\{(\sqrt{2},-\pi,0),(\frac{\sqrt{3}}{2},e,0)\}$

$$(-1,2,0)=a(1,0,0)+b(0,1,0)$$

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13. $(-1,2,3)$, $(4,1,-2)$, $(-14,-1,16)$

assume same plane

$$(-14,-1,16)=a(-1,2,3)+b(4,1,-2)$$

$-14=-a+4b$ ① $-1=2a+b$ ② $16=3a-2b$ ③ verify

$-4=8a+4b$

$-10=-9a$

$a=\frac{10}{9}$

$-1=2\left(\frac{10}{9}\right)+b$

$-\frac{9}{9}-\frac{20}{9}=b$

$b=\frac{-29}{9}$

Verify ③: $LS=16$ $RS=3\left(\frac{10}{9}\right)-2\left(\frac{-29}{9}\right)$

$=\frac{30}{9}+\frac{58}{9}$

$=\frac{88}{9}$

$LS\neq RS$

$\therefore$ the vectors do NOT lie in same plane.

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