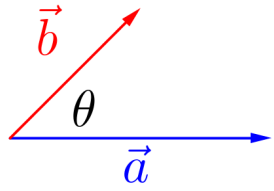
The Dot Product of Geometric Vectors

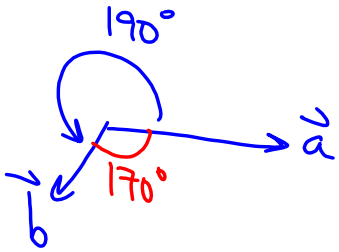
May 9/2018

The dot product is one type of vector multiplication, but the product itself (i.e., the result) is a scalar.



"a dot b"

$$\vec{a} \cdot \vec{b} = |\vec{a}| \left|\vec{b}\right| cos\theta \quad \text{for } 0° \leq \theta \leq 180°$$



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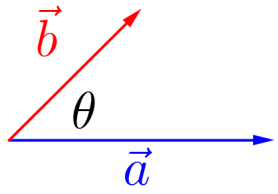
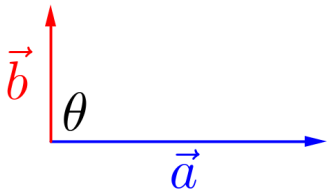
May 7-9:56 PM

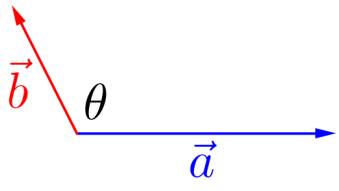


$$0° \leq \theta < 90°$$
$$cos\theta > 0$$
$$\therefore \vec{a} \cdot \vec{b} > 0$$



$$\theta = 90° \quad cos90° = 0$$
$$\therefore \vec{a} \cdot \vec{b} = 0$$

Can be used to test for right angles.



$$90° < \theta \leq 180°$$
$$cos\theta < 0$$
$$\therefore \vec{a} \cdot \vec{b} < 0$$

May 7-10:12 PM

Properties of the Dot Product:

(1) Commutative: $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$

(2) Distributive: $\vec{a} \cdot \left(\vec{b} + \vec{c}\right) = \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c}$

(3) Magnitudes: $\vec{a} \cdot \vec{a} = |\vec{a}|^2$

(4) Associative with scalar:

$$(k\vec{a}) \cdot \vec{b} = \vec{a} \cdot \left(k\vec{b}\right) = k\left(\vec{a} \cdot \vec{b}\right)$$

May 7-10:24 PM

Ex.2 Show that

$$\left(\vec{a} + \vec{b}\right) \cdot \left(\vec{a} - \vec{b}\right) = |\vec{a}|^2 - \left|\vec{b}\right|^2$$

$LS = \vec{a} \cdot \vec{a} + \vec{a} \cdot (-\vec{b}) + \vec{b} \cdot \vec{a} + \vec{b} \cdot (-\vec{b})$

$= |\vec{a}|^2 - \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{b} - \vec{b} \cdot \vec{b}$

$= 0$

$= |\vec{a}|^2 - |\vec{b}|^2$

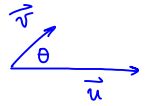
$LS = RS$ ✓

May 14-10:02 PM

Ex.1 Find the angle between vectors **u** and **v** given:

(1) $|\vec{u}| = 3|\vec{v}| \Rightarrow$

(2) $3\vec{u} + \vec{v}$ and $\vec{u} - 8\vec{v}$ are perpendicular.



$(3\vec{u}+\vec{v})\cdot(\vec{u}-8\vec{v}) = 0$

$3|\vec{u}|^2 - 24\vec{u}\cdot\vec{v} + \vec{u}\cdot\vec{v} - 8|\vec{v}|^2 = 0$

$3|\vec{u}|^2 - 23\vec{u}\cdot\vec{v} - 8|\vec{v}|^2 = 0$

$3|\vec{u}|^2 - 23|\vec{u}||\vec{v}|\cos\theta - 8|\vec{v}|^2 = 0$

recall: $|\vec{u}| = 3|\vec{v}| \Rightarrow |\vec{u}|^2 = 9|\vec{v}|^2$

$3(9|\vec{v}|^2) - 23(3|\vec{v}|)|\vec{v}|\cos\theta - 8|\vec{v}|^2 = 0$

$27|\vec{v}|^2 - 69|\vec{v}|^2\cos\theta - 8|\vec{v}|^2 = 0$

$19|\vec{v}|^2 - 69|\vec{v}|^2\cos\theta = 0$

$|\vec{v}|^2(19 - 69\cos\theta) = 0$

$|\vec{v}|^2 = 0$ or $\cos\theta = \frac{19}{69}$

inad $|\vec{v}| > 0$

$\theta = \cos^{-1}\left(\frac{19}{69}\right)$

$\theta \doteq 74°$

don't need cast. S|A

May 14-9:57 PM

Assigned Work

p.377 #2, 5, 6abe, 7acd, 9, 11, 12

May 14-9:22 PM