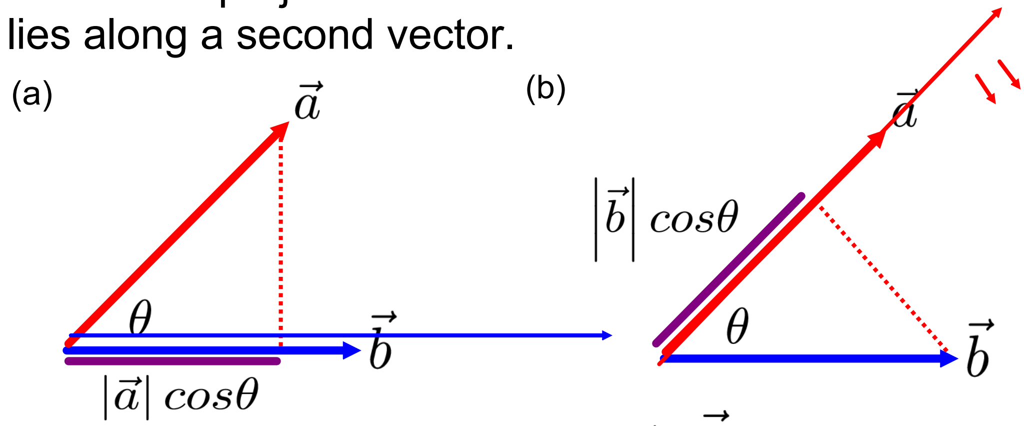
Scalar & Vector Projections

May 11/2018

The scalar projection is a measure of how one vector lies along a second vector.



(a) scalar projection of **a** on **b**: $\dfrac{\vec{a}\cdot\vec{b}}{|\vec{b}|} = |\vec{a}|cos\theta$

(b) scalar projection of **b** on **a**: $\dfrac{\vec{b}\cdot\vec{a}}{|\vec{a}|} = |\vec{b}|cos\theta$

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The vector projection is similar to the scalar projection, but also includes the direction of the second vector.

To include the direction of a vector, but ignore the magnitude, we use the unit vector.

vector projection = $\underbrace{\text{scalar projection}}_{\text{length of shadow}} \times \underbrace{\text{unit vector}}_{\text{direction}}$

vector projection of $\vec{a}$ on $\vec{b}$:

$$\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|} \times \frac{\vec{b}}{|\vec{b}|}$$

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Projections Using Basis Unit Vectors

Given a point in R^3, $P(a, b, c)$, the vector is

$$\overrightarrow{OP} = (a, b, c)$$

We can consider the scalar and vector projections onto the x-, y-, and z-axes, which are trivial:

	scalar projection	vector projection
x:	a	$a\vec{i}$
y:	b	$b\vec{j}$
z:	c	$c\vec{k}$

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Let $\overrightarrow{OP} = (a, b, c)$

From the previous definition of scalar projection:

$$\frac{\overrightarrow{OP} \cdot \vec{i}}{|\vec{i}|} = |\overrightarrow{OP}| cos\alpha = a$$

$$\therefore cos\alpha = \frac{a}{|\overrightarrow{OP}|}$$

Similarly,

$$cos\beta = \frac{b}{|\overrightarrow{OP}|} \quad \text{and} \quad cos\gamma = \frac{c}{|\overrightarrow{OP}|}$$

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$$\cos\alpha = \frac{a}{|\overrightarrow{OP}|} \qquad \cos\beta = \frac{b}{|\overrightarrow{OP}|} \qquad \cos\gamma = \frac{c}{|\overrightarrow{OP}|}$$

These are the direction cosines for $\overrightarrow{OP} = (a, b, c)$ where α, β, and γ are the direction angles $\overrightarrow{OP}$ makes with the positive x-, y-, and z-axes.

OR

α is the angle between $\overrightarrow{OP}$ and x-axis
β " " " " " " y-axis
γ " " " " " " z-axis

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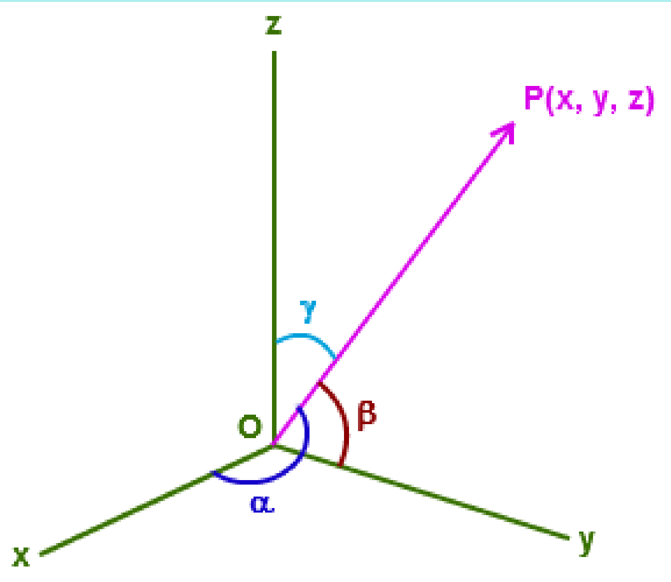

$$\cos\alpha = \frac{a}{|\overrightarrow{OP}|}$$

$$\cos\beta = \frac{b}{|\overrightarrow{OP}|}$$

$$\cos\gamma = \frac{c}{|\overrightarrow{OP}|}$$

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Ex.1 For vectors **a** = (-2, 3, 4) and **b** = (8, 7, -6), find:
(a) the scalar projection of **a** onto **b**.
(b) the scalar projection of **b** onto **a**.
(c) the vector projection of **a** onto **b**.
(d) the direction angles for **a**.

(a) $\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|} = \frac{-16+21-24}{\sqrt{8^2+7^2+6^2}}$

$= \frac{-19}{\sqrt{149}}$

$= \frac{-19\sqrt{149}}{149}$

(b) $\frac{\vec{b}\cdot\vec{a}}{|\vec{a}|} = \frac{-19}{\sqrt{2^2+3^2+4^2}}$

$= \frac{-19}{\sqrt{29}}$

$= \frac{-19\sqrt{29}}{29}$

(c) Vect. $\vec{a}$ on $\vec{b}$ $= \frac{\vec{a}\cdot\vec{b}}{|\vec{b}|} \times \frac{\vec{b}}{|\vec{b}|}$

$= \left(\frac{-19\cancel{\sqrt{149}}}{149}\right)\left(\frac{(8,7,-6)}{\cancel{\sqrt{149}}}\right)$

$= \frac{-19}{149}(8,7,-6)$

$= \left(\frac{-152}{149}, \frac{-133}{149}, \frac{114}{149}\right)$

(d) $\cos\alpha = \frac{a_x}{|\vec{a}|}$ $\quad \cos\beta = \frac{a_y}{|\vec{a}|}$

$\cos\alpha = \frac{-2}{\sqrt{29}}$ $\quad \beta \doteq 56.1°$

$\alpha \doteq 111.8°$ $\quad \gamma \doteq 42.0°$

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Assigned Work

p.398 # 1, 6, 7b, 8, 11, 13, 15, 17

17. prove $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$

$\vec{OP} = (a, b, c)$ let $|\vec{OP}| = \ell$

$\cos\alpha = \frac{a}{|\vec{OP}|} = \frac{a}{\ell}$

$\cos\beta = \frac{b}{\ell}$ $\quad \cos\gamma = \frac{c}{\ell}$

$\sin^2\alpha + \cos^2\alpha = 1$

$\sin^2\alpha = 1 - \cos^2\alpha$

$LS = \sin^2\alpha + \sin^2\beta + \sin^2\gamma$

$= (1-\cos^2\alpha) + (1-\cos^2\beta) + (1-\cos^2\gamma)$

$= 3 - (\cos^2\alpha + \cos^2\beta + \cos^2\gamma)$

$= 3 - \left(\frac{a^2}{\ell^2} + \frac{b^2}{\ell^2} + \frac{c^2}{\ell^2}\right)$

$= 3 - \frac{1}{\ell^2}\underbrace{(a^2+b^2+c^2)}_{|\vec{OP}|^2}$

$= 3 - \frac{1}{\ell^2}(\ell^2)$

$= 2$

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