

Equations of a Line in R3

May 28/2018

(1) Slope-Intercept: $y = mx + b$

No equivalent form in R3.

(2) Vector form: $\vec{r} = \vec{r}_0 + t\vec{m}$

Each vector now has three components.

$$(x, y, z) = (x_0, y_0, z_0) + t(a, b, c)$$

(3) Parametric form:

$$x = x_0 + ta \quad y = y_0 + tb \quad z = z_0 + tc$$

(4) Cartesian form: $Ax + By + Cz + D = 0$

In R3, this form represents a plane, not a line.

$$R2: Ax + By + Cz + D = 0 \quad \vec{n} = (A, B)$$

$$R3: Ax + By + Cz + D = 0 \quad \vec{n} = (A, B, C)$$

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(5) Symmetric form:

The parametric equations all have a common factor 't'.

$$x = x_0 + ta \quad y = y_0 + tb \quad z = z_0 + tc$$

Rearranging each equation for 't' yields:

$$t = \frac{x - x_0}{a} \quad t = \frac{y - y_0}{b} \quad t = \frac{z - z_0}{c}$$

Since the values of 't' are all the same,

$$\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c} \quad a, b, c \neq 0$$

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Ex.1 Determine vector, parametric, and symmetric equations for the line passing through $P(-2, 3, 5)$ and $Q(-2, 4, -1)$.

$$\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP}$$

$$= (0, 1, -6)$$

$$\vec{r} = \vec{r}_0 + t\vec{m}, \quad t \in \mathbb{R}$$

\vec{OP} or \vec{OQ} \vec{PQ}

$$\vec{r} = (-2, 3, 5) + t(0, 1, -6)$$

$$\boxed{x = -2 \quad y = 3 + t \quad z = 5 - 6t}$$

$$\frac{x-x_0}{a} = \frac{y-y_0}{b} = \frac{z-z_0}{c}$$

$$\frac{x+2}{0} = \frac{y-3}{1} = \frac{z-5}{-6}$$

$$x = -2 \text{ and } y - 3 = \frac{z - 5}{-6}$$

parametricsymmetric

Parametric

Symmetric

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Ex.2 Given the symmetric equation, determine vector and parametric equations.

$$\frac{x-3}{5} = \frac{y+2}{3} = \frac{z+5}{7}$$

$$\vec{m} = (5, 3, 7)$$

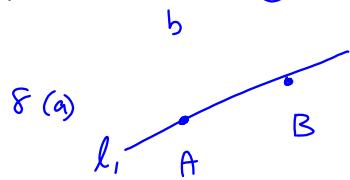
$$x = 3 + 5t \quad y = -2 + 3t \quad z = -5 + 7t$$

$$\vec{r} = (3, -2, -5) + t(5, 3, 7)$$

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Assigned Work:

p.449 # 3, 6, 7, 8, 9, 12, 14



$$\text{show } l_1: \vec{r} = (0, 0, 3) + t(-3, 1, -6)$$

(b) use l_1 to describe \overrightarrow{AB} $t \in \mathbb{R}$

what value of $t \rightarrow A$
 " " of $t \rightarrow B$

$$\overrightarrow{AB}: (0, 0, 3) + t(-3, 1, -6)$$

where $t \in [,]$

$$\begin{cases} x = -3t \\ y = t \\ z = 3 - 6t \end{cases} \quad t \in [,]$$

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14. $l_1: \vec{r}_1 = (4, 4, -3) + t(2, 1, -1)$

$l_2: \vec{r}_2 = (-2, -7, 2) + s(3, 2, -3)$

$$\vec{P_1 P_2} \cdot \vec{r}_1 = 0 \quad \vec{P_1 P_2} \cdot \vec{r}_2 = 0$$

some value of t , $\vec{r}_1 = \vec{OP_1}$
 $= (4+2t_0, 4+t_0, -3-t_0)$
 $\Rightarrow P_1(4+2t_0, 4+t_0, -3-t_0)$

some value of s , $\vec{r}_2 = \vec{OP_2}$

$$P_2(-2+3s_0, -7+2s_0, 2-3s_0)$$

$$\vec{P_1 P_2} = \vec{OP_2} - \vec{OP_1}$$

$$= ()$$

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