Intersection of Lines in R^3

Recall: Intersection of Lines in R^2
(i.e., systems of linear equations)

For two lines in R2, there are three possible outcomes:

(1) One solution (lines intersect at a point).

(2) No solution (lines are parallel, no intersection).

(3) Infinite solutions (lines are coincident).

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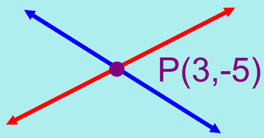
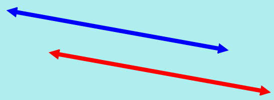
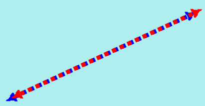
outcome in R^2	algebra	sketch
one solution	result makes sense (e.g., x = 3, y = -5)	
no solution	result not possible (e.g., 0 = 1)	
infinite solutions	result always true (e.g., 3 = 3)	

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Intersection of Lines in R^3

June 1/2018

A linear system may have:

A) One unique solution:
- the lines intersect at one point
- the angle between two lines may be calculated from the dot product of direction vectors

B) No solution:
- the lines do not intersect
- the lines are parallel & distinct (i.e., in the same plane)

OR

- the lines may be skew (i.e., they are not parallel)
- they do not intersect because they lie in different planes

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B) No solution:
- the lines do not intersect
- the lines are parallel & distinct (i.e., in the same plane)

OR

- the lines may be skew (i.e., they are not parallel)
- they do not intersect because they lie in different planes

C) Infinite solutions:
- the lines are coincident

A linear system of two (or more) equations is said to be consistent if it has at least one solution, otherwise it is inconsistent (no solutions).

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Equations of Lines in R^2:

slope-intercept $y = mx + b$

cartesian $Ax + By + C = 0$

vector $\vec{r} = \vec{r}_0 + t\vec{m} \qquad t \in \mathbb{R}$

parametric
$$x = x_0 + tm_x$$
$$y = y_0 + tm_y \qquad t \in \mathbb{R}$$

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Equations of Lines in R^3:

vector $\vec{r} = \vec{r}_0 + t\vec{m} \qquad t \in \mathbb{R}$

parametric
$$x = x_0 + tm_x$$
$$y = y_0 + tm_y \qquad t \in \mathbb{R}$$
$$z = z_0 + tm_z$$

symmetric
$$\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}$$
$$a, b, c \neq 0$$

$\vec{d} = (a, b, c)$

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Ex.1 Solve the following systems and describe the geometric relationship between the lines.

(a) $\vec{r}_1 = (-2, 0, -3) + t(5, 1, 3)$
$\vec{r}_2 = (5, 8, -6) + s(-1, 2, -3)$

$-2 + 5t = 5 - s$ ①
$0 + t = 8 + 2s$ ②
$-3 + 3t = -6 - 3s$ ③

3 equations, 2 unknowns. Solve 2 and verify 3rd.

① $-2 + 5t = 5 - s$ ② $t = 8 + 2s$

$-2 + 5(8 + 2s) = 5 - s$

$-2 + 40 + 10s = 5 - s$ $t = 8 + 2(-3)$

$11s = -33$ $t = 2$

$s = -3$

③ $LS = -3 + 3t$ $RS = -6 - 3s$
$= -3 + 3(2)$ $= -6 - 3(-3)$
$= 3$ $= 3$

$LS = RS$ ✓

find POI using s, t

$\vec{r}_1 = (-2, 0, -3) + 2(5, 1, 3)$
$= (8, 2, 3)$

∴ POI is $P(8, 2, 3)$ (non-parallel lines) same plane

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(b) $\frac{x+4}{3} = \frac{y-12}{4} = \frac{z-3}{6}$ ℓ_1

$\frac{x}{\frac{1}{2}} = \frac{y-10}{\frac{2}{3}} = z + 5$ ℓ_2

recall: $\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}$

where $\vec{d} = (a, b, c)$

$\ell_1: \vec{r}_1 = (-4, 12, 3) + s(3, 4, 6)$
$\ell_2: \vec{r}_2 = (0, 10, -5) + t(\frac{1}{2}, \frac{2}{3}, 1)$

① could solve system as usual
② recognize that $(\frac{1}{2}, \frac{2}{3}, 1) \times 6 = (3, 4, 6)$ parallel direction vectors

→ check that a point from ℓ_1 is on ℓ_2 ($\vec{r}_2$)
$(-4, 12, 3)$

$-4 = 0 + \frac{1}{2}t$ $12 = 10 + \frac{2}{3}t$ $3 = -5 + t$
$-8 = t$ $3 = t$ $8 = t$

∵ t is inconsistent

∴ no solution, lines are parallel but distinct.

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Assigned Work:

p.497 # 8, 9, 11, 12

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