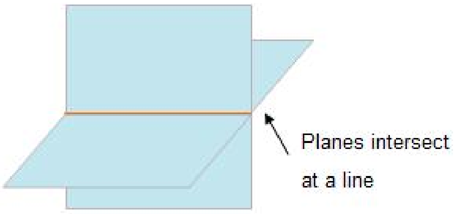
Intersections of Two Planes

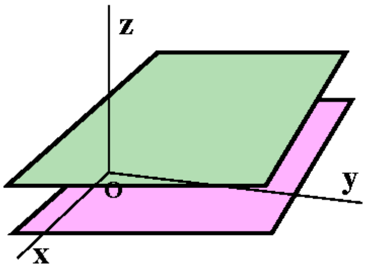
June 5/2018

(1) If two planes intersect along a line, the system has an infinite number of solutions, as described by the parametric equations of the line.



(2) If two planes are coincident (i.e., same plane), the system has an infinite number of solutions, as described by either of the two given equations of the plane.

(3) If two planes are parallel (i.e., their normal vectors are parallel) and distinct, the system has no solution.



May 31-12:33 PM

Ex.1 Solve each system and give a geometric description of the planes.
(i.e., line intersection, coincident, parallel & distinct)

a) $x + 4y - 3z + 6 = 0$ ①
$2x + 8y - 6z + 11 = 0$ ②

$\vec{n}_1 = (1, 4, -3)$
$\vec{n}_2 = (2, 8, -6)$

①×2: $2x + 8y - 6z + 12 = 0$

$0x + 0y + 0z - 1 = 0$

$-1 = 0$ inconsistency

∴ no solution, planes are parallel but distinct.

May 31-1:02 PM

b) ① $5x - y + 2z - 9 = 0$
② $-25x + 5y - 10z + 45 = 0$

$\vec{n}_1 = (5, -1, 2)$
$\vec{n}_2 = (-25, 5, -10)$

①×5: $25x - 5y + 10z - 45 = 0$

$0 = 0$

∴ infinite solutions, planes are coincident (same)

May 31-1:08 PM

c) $4x + 7y - 33z + 17 = 0$ ①
$8x + 5y - 3z + 7 = 0$ ②

$\vec{n}_1 = (4, 7, -33)$
$\vec{n}_2 = (8, 5, -3)$
not collinear
∴ planes not parallel
∴ intersection is line

2×①: $8x + 14y - 66z + 34 = 0$
②: $8x + 5y - 3z + 7 = 0$

$9y - 63z + 27 = 0$ ÷9
$y - 7z + 3 = 0$
$y = 7z - 3$

for any z, this equation tells us y

form a parametric equations

let $z = t$
$y = 7t - 3$

$8x + 5(7t - 3) - 3t + 7 = 0$
$8x + 35t - 15 - 3t + 7 = 0$
$8x = -32t + 8$
$x = -4t + 1$

or: $\vec{r} = (1, -3, 0) + t(-4, 7, 1)$

May 31-1:13 PM

Assigned Work

p.516 # 1, 2, 3, 6, 8, 10

May 31-1:31 PM