Intersections of Three Planes

June 12/2018

When presented with three planes, it is important to consider the equations and normals, as well as performing the algebra to solve the system.

Consistent systems (see p.520-521):

Case 1: A point of intersection. All normal vectors different.
(three normal vectors do NOT form a plane)

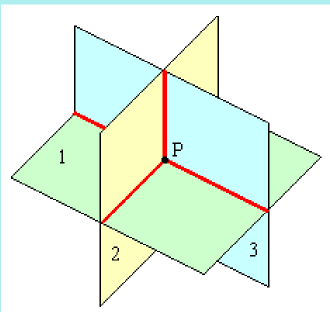
Case 2a: A line of intersection. All normal vectors different.
(three normal vectors lie in the same plane)

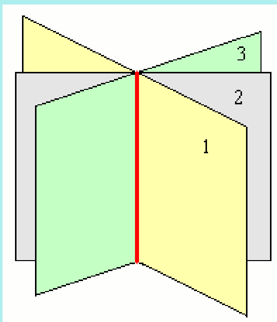
Case 2b: A line of intersection. Two planes coincident.

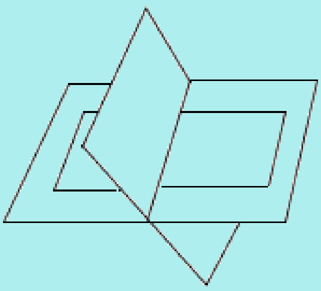
Case 3: A plane of intersection. All planes the same.

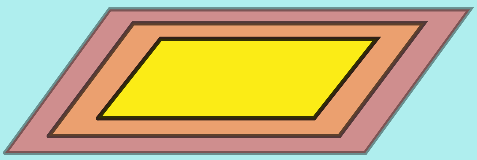
Jun 4-8:39 AM

Consistent Systems:









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Inconsistent Systems (see p.526-527):
(all have no solution)

Case 1: Each plane intersects another plane along a line, but none of the lines are the same.

Case 2: There are two lines of intersection, meaning two planes are parallel (but distinct), and each of them intersects the third plane at a line.

Case 3: Three planes are parallel and distinct, so there is no intersection between any plane.

Case 4: Two planes are coincident, while the third is parallel but distinct. There are infinite solutions for the first two (i.e., a plane of intersection), but nothing in common with the third.

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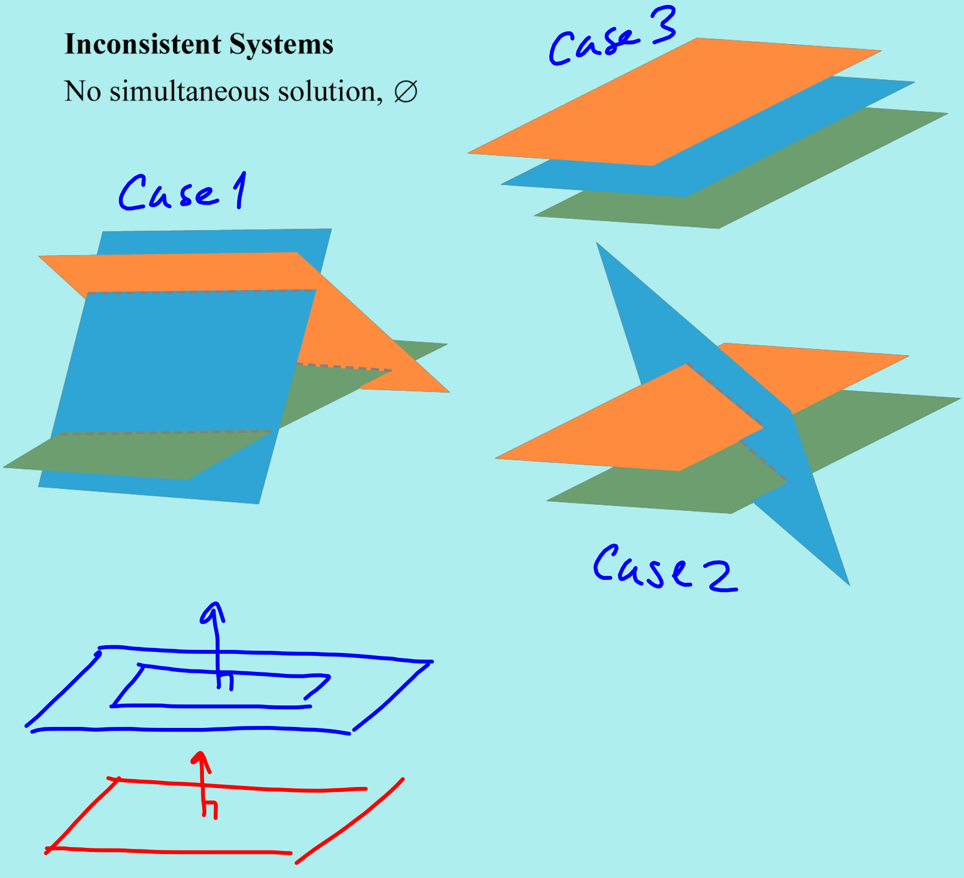
Inconsistent Systems

No simultaneous solution, ∅



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Ex.1 Solve each system (if possible) and give a geometric description of the system.

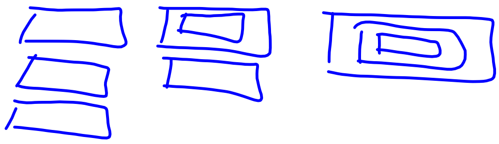
(a) $\left[\begin{array}{ccc|c} 2 & -1 & 3 & 2 \\ 4 & -2 & 6 & 3 \\ 6 & -3 & 9 & 4 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 1 & \frac{-1}{2} & \frac{3}{2} & \frac{2}{3} \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right]$

(1) normals
(2) solution options
(3) solve

$2x - y + 3z = 2$ $\quad \vec{n}_1 = (2,-1,3)$
$4x - 2y + 6z = 3$ $\quad \vec{n}_2 = (4,-2,6)$ ×2
$6x - 3y + 9z = 4$ $\quad \vec{n}_3 = (6,-3,9)$ ×3

① normals are collinear (parallel planes)

②



③ no solution ✓

→ looking in more detail, we can show planes are all distinct.

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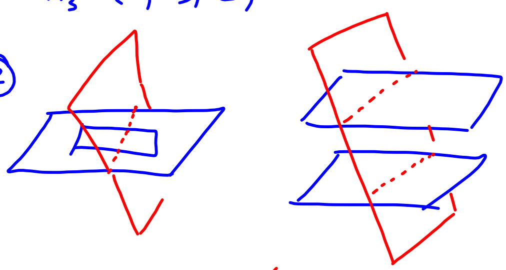
(b) $\left[\begin{array}{ccc|c} 2 & -1 & 3 & 2 \\ 4 & -2 & 6 & 3 \\ 1 & -3 & 2 & -10 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 1 & \frac{-1}{2} & \frac{3}{2} & \frac{2}{3} \\ 0 & 1 & \frac{-1}{5} & 1 \\ 0 & 0 & 0 & 1 \end{array}\right]$

(1) normals
(2) solution options
(3) solve

① $\vec{n}_1 = (2,-1,3)$ ×2 } collinear
$\vec{n}_2 = (4,-2,6)$
$\vec{n}_3 = (1,-3,2)$

②



③ no solution ✓

→ two parallel but distinct planes with a third plane crossing each of them at a different line.

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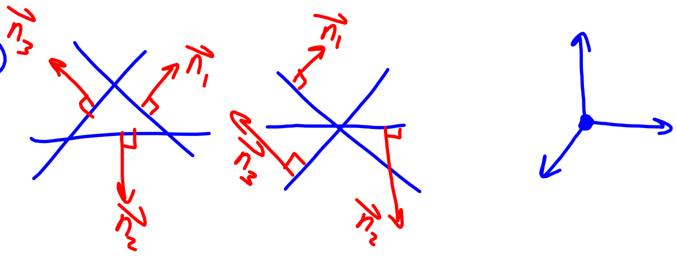
(c) $$\left[\begin{array}{ccc|c} 2 & -1 & 3 & 2 \\ 1 & -3 & 2 & -10 \\ 5 & -5 & 8 & -3 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & \frac{7}{5} & 0 \\ 0 & 1 & \frac{-1}{5} & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$$

(1) normals
(2) solution options
(3) solve

① $\vec{n}_1 = (2,-1,3)$
$\vec{n}_2 = (1,-3,2)$
$\vec{n}_3 = (5,-5,8)$

} all different, non-collinear

②



③ no solution ✓

→ triangular prism formed

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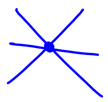
(d) $$\left[\begin{array}{ccc|c} 2 & -1 & 3 & 2 \\ 1 & -3 & 2 & -10 \\ 5 & -5 & 8 & -6 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & \frac{7}{5} & \frac{16}{5} \\ 0 & 1 & \frac{-1}{5} & \frac{22}{5} \\ 0 & 0 & 0 & 0 \end{array}\right]$$

(1) normals
(2) solution options
(3) solve

① $\vec{n}_1 = (2,-1,3)$
$\vec{n}_2 = (1,-3,2)$
$\vec{n}_3 = (5,-5,8)$

} non-collinear

②



③ infinite solutions (line)

$y - \frac{1}{5}z = \frac{22}{5}$

$5y - z = 22$

$5y - 22 = z$

let $\boxed{y = t}$

$\boxed{z = 5t - 22}$

$x + \frac{7}{5}z = \frac{16}{5}$

$x + \frac{7}{5}(5t - 22) = \frac{16}{5}$

$x + 7t - \frac{154}{5} = \frac{16}{5}$

$\boxed{x = -7t + 34}$

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Assigned Work:

p.532 #9, 10, 13

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