The Rule of Sum

Sept 20/2018

When counting outcomes of multiple events together, we must consider how they relate to each other.

(1) Fundamental Counting Principle
- used when one event AND another occur together
- product rule for independent events:

$$n(A \cap B) = n(A \text{ and } B) = n(A) \times n(B)$$

(2) Rule of Sum
- used when one event OR another occurs.

$$n(A \cup B) = n(A \text{ or } B) = n(A) + n(B) - n(A \cap B)$$

Note: For mutually exclusive events $n(A \cap B) = 0$

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Ex. A yearbook cover can show 7 students from a single team or club. The Rugby 7's team has 10 members, and student's council has 7 members. How many ways can students be arranged on the cover?

Note: Assume there is no overlap between them.

$$n(\text{cover}) = n(\text{rugby}) + n(\text{SC}) - \underbrace{n(R \cap SC)}_{=0}$$

$$= {}_{10}P_7 + {}_7P_7$$

$$= \frac{10!}{(10-7)!} + 7!$$

$$= \frac{10!}{3!} + 7!$$

$$= 609840$$

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Ex. Three players are dealt one card each. How many ways could the cards be face cards OR red cards?

Note: Since each player gets a different card, order matters.

$n(FC) = {}_{12}P_3$ # of ways to deal everybody a face card

12 FC deal 3 cards

$n(R) = {}_{26}P_3$

overlap between FC and R

$n(FC \cap R) = {}_{6}P_3$ reds cards only

$n(FC \text{ or } R) = n(FC) + n(R) - n(FC \text{ and } R)$

only FC of any suit

$= {}_{12}P_3 + {}_{26}P_3 - {}_{6}P_3$

$= \frac{12!}{9!} + \frac{26!}{23!} - \frac{6!}{3!}$

$= 16800$

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A <u>direct method</u> involves counting the desired events. When this is very complicated, consider an <u>indirect method</u> where unwanted events are subtracted from the total number of events.

Ex. Ten members of the hockey team line up to receive medals. How many ways can this be done if:
(a) there is no restriction.
(b) the captain and assistant must sit together.
(c) the captain and assistant must **not** sit together.

(a) $10 \times 9 \times 8$ _ _ _ _ _ _ _ _ _ _ $10!$

(b) $\underbrace{\boxed{\_\ \_}_{C \cap A,\ 2!}\ \_\ \_\ \_\ \_\ \_\ \_\ \_\ \_}_{9!}$ $9!\,2!$

(c) indirect method:

$10! - 9!\,2! = 2903040$

total possible arrangements; possible w $C \cap A$; # possible with $(C \cap A)'$

Recall: $P(A) = 1 - P(A')$

OR

$P(A) + P(A') = 1$

$n(A) + n(A') = n(\text{all})$

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Ex. How many ways could the letters in $\underbrace{\text{FACTOR}}_{6}$ be arranged so that:
(a) vowels are **not** together.
(b) consonants are **not** together.
(c) vowels are together **and** consonants are together.
(d) vowels are together **or** consonants are together.

(a) try vowels together (indirect)

$$\underbrace{\underbrace{\boxed{\_\ \_}}_{\text{vowels } 2!}\ \_\ \_\ \_\ \_}_{5!}$$

$n(\text{V together}) = 2!5!$

$n(\text{all}) = 6!$

$n\left(\begin{smallmatrix}\text{vowels}\\ \text{apart}\end{smallmatrix}\right) = 6! - 2!5!$

(b)

$$\underbrace{\underbrace{\boxed{\_\ \_\ \_\ \_}}_{4!}\ \_\ \_}_{3!}$$

$n(\text{C together}) = 4!3!$

$n(\text{all}) = 6!$

$n(\text{C apart}) = 6! - 4!3!$

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(c) $n(\text{V tog.} \cap \text{C tog.}) = 2!2!4!$

$$\underbrace{\underbrace{\boxed{\_\ \_}}_{2!}\ \underbrace{\boxed{\_\ \_\ \_\ \_}}_{4!}}_{2!}$$

$= 96$

(d) $n(\text{V tog. OR C tog.})$

$n(A \cap B) + n(A \cup B) = n(A) + n(B)$ RECALL

$n(A \cap B) = n(A) + n(B) - n(A \cup B)$

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$n(\text{V tog.} \cup \text{C tog.}) = 240 + 144 - 96$

$= 288$

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Assigned Work:

p.86 #5, 7, 9a, 10, 11 15, 18

5. 1, 2, 3, 4, 5

even number : divisible by 2

→ ends in 0, 2, 4, 6, 8

$n(\text{even}) = 1 \times 2 \times 3 \times 4 \times 2$

2 or 4

end w/ 2   1 $\cancel{2}$ 3 4 5

$4 \times 3 \times 2 \times 1\ (2) + 4 \times 3 \times 2 \times 1\ (4)$

$n(\text{even})_{\text{5-digit}} = {}_2P_1 \times {}_4P_4$

2 even to choose from, use one at end

$= (2)(4!)$

$= 48$

$n(\text{even}) = n\left(\begin{smallmatrix}\text{even}\\ \text{5 dig}\end{smallmatrix}\right) + n\left(\begin{smallmatrix}\text{even}\\ \text{4 dig}\end{smallmatrix}\right) + n(E3D) + n(E2D) + n(E1D)$

$= {}_2P_1 \times {}_4P_4 + {}_2P_1 \times {}_4P_3 + {}_2P_1 \times {}_4P_2 + {}_2P_1 \times {}_4P_1 + {}_2P_1$

end w/ 2 or 4   use 4 digits

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10. 5-digits include 4, 6, or both

① direct method

→ count all outcomes that match criteria

10 choices per digit

9 choices because 0 makes a 4-digit number

no overlap accounted for yet.

4/6

$2 \times 10 \times 10 \times 10 \times 10$

4/6

$+ 9 \times 2 \times 10 \times 10 \times 10$

$+ 9 \times 10 \times 2 \times 10 \times 10$

$+ 9 \times 10 \times 10 \times 2 \times 10$

$+ 9 \times 10 \times 10 \times 10 \times 2$

② indirect method

→ focus on what you don't want

$n(A) = n(S) - n(A')$

A' : a number with no fours, sixes

$7 \times 8 \times 8 \times 8 \times 8$

$= 28672$

S:

$9 \times 10 \times 10 \times 10 \times 10$

$= 90000$

$n(A) = 90000 - 28672$

or

$P(A) = \dfrac{n(A)}{n(S)}$

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15. mutually exclusive

$n(6\text{-digit}) + n(7d) + n(8d)$

1

(a) no restriction (repeat chars)

$\_\ \_\ \_\ \_\ \_\ \_\ \_$

$62 \times 62 \times \cdots$

$= (62)^6$

$n(6 \cup 7 \cup 8)$

$= 62^6 + 62^7 + 62^8$

(b) no repetition, order matters

$\_\ \_\ \_\ \_\ \_\ \_$

$62 \times 61 \times 60 \times 59 \times 58 \times 57$

$_{62}P_6 = \frac{62!}{56!}$

$+$

$_{62}P_7$

$+$

$_{62}P_8$

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