Probability Problems using Permutations
(except for when we don't...)

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Solving Probability Problems

Sept 25/2018

Ex. A 10-question multiple choice test has possible answers A, B, C, or D. What is the probability that:
(a) C is correct for all questions?
(b) C is never used?

(a) $P(C\ only) = \left(\frac{1}{4}\right)\left(\frac{1}{4}\right)\left(\frac{1}{4}\right)\cdots\left(\frac{1}{4}\right)$

Q1 Q2 Q3 ⋯ Q10

$= \left(\frac{1}{4}\right)^{10}$

independent outcomes, probability for each does not change

OR

$n(C\ only) = 1 \times 1 \times \cdots \times 1$

$= (1)^{10}$

$= 1$

$P(C\ only) = \frac{1}{4^{10}}$

$n(all) = (4)^{10}$

(b) $n(C') = (3)^{10}$

$P(C') = \frac{3^{10}}{4^{10}} = \left(\frac{3}{4}\right)^{10}$

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Ex. Eight people are on a waiting list for concert tickets. What is the probability that they will be chosen from youngest to oldest?

$n(\text{arrangements}) = {}_8P_8$

$n(S)$

$= 8!$

$n(A)$

$n(\text{order of age}) = 1$

$P(\text{in order of age}) = \frac{1}{8!}$

$P(A)$

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Ex. Three cards are selected without replacement. What is the probability of:
(a) a king, then two queens (KQQ), in order?
(b) a king and two queens, in any order?

(a) $52 \times 51 \times 50 = {}_{52}P_3 = n(all)$

$= 132600$

① draw K: $4 = {}_4P_1$

② draw Q: $4 = {}_4P_1$

③ draw Q: $3 = {}_3P_1$

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$n(KQQ) = (4)(4)(3)$

$= {}_4P_1 \times {}_4P_2$

$= 48$

$P(KQQ) = \frac{48}{132600}$

$= \frac{2}{5525}$

(b) $n(KQQ) + n(QKQ) + n(QQK)$

$= ({}_4P_1)({}_4P_2) + ({}_4P_1)({}_4P_1)({}_3P_1) + ({}_4P_2)({}_4P_1)$

$= 48 + 48 + 48$

$= 144$

$P(KQQ \cup QKQ \cup QQK) = \frac{144}{132600}$

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Ex. From a class of 30 students, what is the probability of:
(a) no two people have the same birthday?
(b) at least two people have the same birthday?

(a) $1, 2, 3, 4, \ldots, 29, 30$
$2, 3, 4, \ldots, 29, 30, 31$

$n(\text{no birthdays same}) = {}_{365}P_{30} = n(A)$
365: # days in year; 30: lists of 30 days, no repetition

$n(\text{all}) = 365 \times 365 \times 365 \times \ldots \times 365 = (365)^{30}$
all ways to have birthdays, repetition allowed

$P(A) = \dfrac{{}_{365}P_{30}}{(365)^{30}}$ (no birthdays same)
$\doteq 0.293$
$\doteq 29.3\%$

(b) $P(A') = 1 - P(A)$

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Assigned Work:

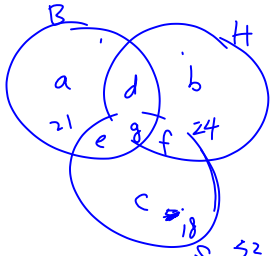
p.93 # 1 - 4 (warm up, if needed)
5, 6, 8, 9, 10, 13, 14, 17, 19, 21
20*, 23*

${}_nP_r = \dfrac{n!}{(n-r)!}$ — A — — — —

$D(n) = n! \sum_{r=0}^{n} \dfrac{(-1)^r}{r!}$

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap C) - n(A \cap B) - n(B \cap C) + n(A \cap B \cap C)$



$n(B) = 24 = a + e + d + g$
$n(B \cap H) = d + g$

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14. Five people each choose a card from a standard deck. They replace the card after making their choice.

a) What are the odds against at least two people choosing the same card?

b) What are the odds against at least two people choosing the same denomination?

* like birthday problem

(a) $n(all) = (52)^5$

A = at least two people pick the same card

A' = nobody picks same card

$n(A') = {}_{52}P_5$ $52 \times 51 \times 50 \times 49 \times 48$ # of ways to draw different cards.

$n(A) = n(all) - n(A')$

(a) $P(A') : P(A)$ $P(A) = \frac{n(A)}{n(all)}$

$P(A') = \frac{n(A')}{n(all)}$

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