Permutations with Non-Ordered Elements Sept 27/2018

Consider possible arrangements of:

aAB	AAB
aAB aAB BaA aBA ABa BAa	AAB ~~AAB~~ BAA ABA ~~ABA~~ ~~BAA~~
$n(S_1)=6$	$n(S_2)=3$
aABC	AABC
$n(S_3)={}_4P_4$ $=4!$ $=24$	$n(S_4)=12$

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aAB	AAB
aAB aAB BaA aBA ABa BAa	AAB ABA BAA
n(all) = 6	n(all) = 3

The reason for the difference is that aA has 2! permutations, which are lost when we switch to AA.

$$n(aAB) = {}_3P_3 = 3! \qquad n(AAB) = \frac{{}_3P_3}{{}_2P_2} = \frac{3!}{2!}$$

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Ex. Compare possible arrangements for:

(a) A_1A_2BC and AABC
(b) $A_1A_2B_1B_2$ and AABB

(a) $S_1 = A_1A_2BC$ $S_2 = AABC$

$n(S_1) = {}_4P_4 = 4!$ $n(S_2) = \frac{{}_4P_4}{{}_2P_2}$

permutations of AA that are identical

(b) $S_3 = A_1A_2B_1B_2$ $S_4 = AABB$

$n(S_3) = {}_4P_4 = 4!$ $n(S_4) = \frac{{}_4P_4}{{}_2P_2 \times {}_2P_2}$

AA BB

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In general, the number of arrangements of n-elements is given by:

$$ {}_nP_n = n! $$

For p duplicate elements:

$$ \frac{{}_nP_n}{p!} = \frac{n!}{p!} $$

If there are multiple sets of duplicate elements (p, q, r, ...)

$$ \frac{n!}{p!q!r!\ldots} $$

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Ex. A hockey team ends the season with 12 wins, 8 losses and 4 ties. How many ways could this occur?

$$n(all) = \frac{{}_{24}P_{24}}{12!\,8!\,4!}$$

← ordering of games vs unique teams

the teams we beat don't matter

WWWW...WW

12!

Who we lost to doesn't matter

LLL...LL

8!

TTTT

4!

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Ex. For the letters in the word NUMBER, how many ways to arrange if:
(a) the consonants must stay in the same order.
(b) the vowels must stay in the same order.
(c) ordered consonants and ordered vowels.

(a) 6 letters, $n(all) = 6! = {}_6P_6$

NMBR

NMUBRE
NUMBER
MUBREN
BURNEM

4! ways to arrange NMBR

Since NMBR must stay in order, it is like CCCC

NUMBER → CUCCEC

$$\frac{6!}{4!}$$

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(b) NUMBER → NVMBVR

vowel

$n(\text{vowel in order}) = \frac{6!}{2!}$

(c) NUMBER → CVCCVC

$n\begin{pmatrix} \text{C in order} \\ \text{V in order} \end{pmatrix} = \frac{6!}{4!\,2!}$

remove arrangements of NMBR

remove arrangements of UE

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Assigned Work:

p.108 # 4, 5
6, 9, 11, 13, 14, 15
18, 19

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