Dividing Polynomials

Sept 27/2016

Ex.1 What is $107 \div 4$?
recall: long division!

$$
\begin{array}{r}
26 \\
4\overline{)107} \\
-\ 8 \\
\hline
27 \\
-\ 24 \\
\hline
3
\end{array}
$$

3 → Remainder

$107 \div 4 = 26 \text{ R } 3$

$= 26 \frac{3}{4}$

$= 26 + \frac{3}{4}$

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Ex.2 Determine the quotient and remainder for

$$(3x^3 - 5x^2 - 7x - 1) \div (x - 3)$$

$$
\begin{array}{r}
3x^2 + 4x + 5 \\
x-3\overline{)3x^3 - 5x^2 - 7x - 1} \\
3x^3 - 9x^2 \\
\hline
4x^2 - 7x \\
4x^2 - 12x \\
\hline
5x - 1 \\
5x - 15 \\
\hline
14
\end{array}
$$

① $3x^2(x-3)$
$= 3x^3 - 9x^2$

$$\frac{3x^3 - 5x^2 - 7x - 1}{x-3} = 3x^2 + 4x + 5 + \frac{14}{x-3}$$

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Ex.3 Use synthetic division (see p.164 for more detail)

$$(3x^3 - 5x^2 - 7x - 1) \div (x - 3)$$

$x - k$

$k = 3$

k

$$\begin{array}{r|rrrr} 3 & 3 & -5 & -7 & -1 \\ & & 9 & 12 & 15 \\ \hline & 3 & 4 & 5 & 14 \rightarrow R \end{array}$$

$x^2 \quad x \quad \text{const}$

$$= 3x^2 + 4x + 5 + \frac{14}{x-3}$$

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Notes on synthetic division:

(1) The divisor must be in the form $(x - k)$

(2) All terms must be represented, even if they have a coefficient of zero

(3) If the root, k, is a fraction, you may prefer long division to reduce the risk of an error.

(4) If the remainder of the division is zero, then both the quotient and the divisor are <u>factors</u> of the original polynomial.

$\div (3x - 2)$

$\div \left[3\left(x - \frac{2}{3}\right)\right]$

$$\begin{array}{r|l} \frac{2}{3} & \\ \hline & \end{array}$$

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Ex. Given (3x+2) is a factor, determine other factors of $6x^3 + 19x^2 + x - 6$ using synthetic division.

$$\begin{array}{r|l} & 2x^2 + 5x - 3 \\ 3x+2 & 6x^3 + 19x^2 + x - 6 \\ & \underline{6x^3 + 4x^2} \\ & 15x^2 + x \\ & \underline{15x^2 + 10x} \\ & -9x - 6 \\ & \underline{-9x - 6} \\ & 0 \rightarrow R \end{array}$$

$3x+2$
$= 3(x + \frac{2}{3})$

$$\begin{array}{c|cccc} -\frac{2}{3} & 6 & 19 & 1 & -6 \\ & \downarrow & -4 & -10 & 6 \\ \hline & 6 & 15 & -9 & 0 \end{array}$$

$$\frac{6x^3 + 19x^2 + x - 6}{3(x + \frac{2}{3})} = \frac{6x^2 + 15x - 9}{3}$$

$$= 2x^2 + 5x - 3$$

$$6x^3 + 19x^2 + x - 6 = (3x+2)(2x^2 + 5x - 3)$$

S: 5
P: -6
I: 6, -1

$$= (3x+2)(2x^2 + 6x - x - 3)$$
$$= (3x+2)(2x(x+3) - 1(x+3))$$
$$= (3x+2)(x+3)(2x-1)$$

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p.168 # 1, 4*, 5ace, 6ace, 7ac, 8d, 9ac, 10ace, 11, 12, 14

10(e)

$$\begin{array}{r|l} & 1x^5 \quad\quad\quad\quad + 3x + \frac{2}{3} \\ 3x+5 & 3x^6 + 5x^5 + 0x^4 + 0x^3 + 9x^2 + 17x - 1 \\ & \underline{3x^6 + 5x^5} \\ & 0 \quad 0 \quad 0 \quad 9x^2 + 17x \\ & \underline{9x^2 + 15x} \\ & 2x - \cancel{1}\tfrac{3}{3} \\ & \underline{2x + \frac{10}{3}} \\ & R \quad -\frac{13}{3} \end{array}$$

$(k)(3x) = 2x$

$k = \frac{2\cancel{x}}{3\cancel{x}}$

$k = \frac{2}{3}$

$3x+5$
$= 3\left(\frac{3x+5}{3}\right)$
$= 3(x + \frac{5}{3})$
$a = -\frac{5}{3}$

$f(-\frac{5}{3}) = \frac{-13}{3}$

Oct 1-9:36 AM

p.168 # 1, 4*, 5ace, 6ace, 7ac, 8d, 9ac, 10ace, 11, 12, 14

4. $$\frac{f(x)}{D} = Q + \frac{R}{D}$$

(b) $$\frac{f(x)}{2x+4} = 3x^3 - 5x + 8 - \frac{3}{2x+4}$$

$$f(x) = (2x+4)\left(3x^3 - 5x + 8 - \frac{3}{2x+4}\right)$$

(c) $$D\left[\frac{6x^4+2x^3+3x^2-11x-9}{D}\right] = \left[2x^3+x-4-\frac{5}{D}\right] \times D$$

$$6x^4+2x^3+3x^2-11x-9 = D(2x^3+x-4) - 5$$

$$\frac{6x^4+2x^3+3x^2-11x-4}{2x^3+x-4} = \frac{D(2x^3+x-4)}{2x^3+x-4}$$

$$= D$$

$$\begin{array}{r|l} & 3x+1 \\ \hline 2x^3+0x^2+x-4 & 6x^4+2x^3+3x^2-11x-4 \\ & \underline{6x^4+0x^3+3x^2-12x} \downarrow \\ & 2x^3+0x^2+x-4 \end{array}$$

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6(e)

$$\begin{array}{r|rrrrr} & 12 & -56 & 59 & 9 & -18 \\ -\frac{1}{2} & \downarrow & -6 & 31 & -45 & 18 \\ \hline & 12 & -62 & 90 & -36 & 0 \end{array}$$

$$2x+1 = 2\left(x+\frac{1}{2}\right)$$

$$k = -\frac{1}{2}$$

$$\frac{12x^4-56x^3+59x^2+9x-18}{2\left(x+\frac{1}{2}\right)}$$

$$= \frac{12x^3-62x^2+90x-36}{2}$$

$$= 6x^3-31x^2+45x-18$$

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7\. (c) $$\frac{f(x)}{5x+2} = x^3+4x^2-5x+6+\frac{x-2}{5x+2}$$

$$f(x) = (5x+2)(x^3+4x^2-5x+6)+x-2$$

$$=$$

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11\. $V = l \times w \times h$

$$h = \frac{V}{lw}$$

$$= \frac{x^3+6x^2+11x+6}{(x+2)(x+3)}$$

①: $\frac{V}{l}$ = answer

$\frac{\text{answer}}{w} = h$

②: expand lw = quadratic

$$h = \frac{V}{\text{quadratic}}$$

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$$\begin{array}{r} 4x^2 + 3x - 5 \\ 2x+1 \overline{) 8x^3 + 10x^2 - px - 5} \\ \underline{8x^3 + 4x^2} \\ 6x^2 - px \\ \underline{6x^2 + 3x} \\ (-p-3)x - 5 \\ \underline{-10x - 5} \\ 0 + 0 \end{array}$$

$-px - 3x$

$(-p-3)x$

can only occur if

$-p-3 = -10$

$-p = -7$

$p = 7$

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14.

$x-1 \overline{) x^2 - 1}$

$x-1 \overline{) x^3 - 1}$

$x-1 \overline{) x^4 - 1}$

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