Factoring Polynomials

Oct 1/2018

Remainder Theorem: When a polynomial, $f(x)$, is divided by $x-a$, the remainder is equal to $f(a)$.

Factor Theorem:

If the remainder, or $f(a)$, is equal to zero, then $x-a$ is a factor of the polynomial $f(x)$.

For example, recall:

$$\frac{3x^3-5x^2-7x-1}{x-3}=3x^2+4x+5+\frac{14}{x-3}$$

$f(x)=3x^3-5x^2-7x-1$

$a=3$

$f(3)=3(3)^3-5(3)^2-7(3)-1$

$=3(27)-5(9)-21-1$

$=81-45-22$

$=14$

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Ex.1 Use the factor theorem to determine one factor of

$f(x)=x^3+4x^2+x-6$

then completely factor the function.

$x-1$: $f(1)=1+4+1-6$

$=0$ ✓

$x+1$: $f(-1)=-1+4-1-6$

$=-4$ ✗

$$\begin{array}{r|l} & x^2+5x+6 \\ \hline x-1 & x^3+4x^2+x-6 \\ & \underline{x^3-x^2} \\ & \quad 5x^2+x \\ & \quad \underline{5x^2-5x} \\ & \qquad 6x-6 \\ & \qquad \underline{6x-6} \\ & \qquad\quad 0 \end{array}$$

$f(x)=(x-1)(x^2+5x+6)$

$=(x-1)(x+2)(x+3)$

S: 5 M: 6

P: 6 A: 5

I: 2,3 N: 2,3

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A rational number can be expressed as a fraction with an integer numerator and denominator (but no division by zero).

$$\frac{a}{b} \text{ where } a,b \in \mathbb{Z}, b \neq 0$$

A rational root is a zero which is a rational number. For a polynomial, roots can be expressed as factors:

$$\left(x - \frac{a}{b}\right) \text{, or, more commonly, } (bx - a)$$

$$= b\left(\frac{bx-a}{b}\right)$$

$$= b\left(x - \frac{a}{b}\right)$$

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The rational roots test allows us to limit our search for roots using the leading term and the constant (last) term.

For a polynomial in the form

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_1 x + a_0$$

the possible rational roots are

$$\frac{\text{all factors of constant term}}{\text{all factors of leading coefficient}}$$

note: Some roots are irrational, and there is no guarantee that the rational root test will be successful.

Therefore, not all polynomials will be factorable.

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For example, $y = 3x^2 + 10x - 8$ has a constant term -8 and a leading coefficient of 3.

factors of 8 are 1, 2, 4, 8 $(-1)(-8)$ $(-2)(-4)$
factors of 3 are 1, 3

possible rational roots are $\dfrac{\pm 1, 2, 4, 8}{1, 3}$

As a list: $\pm 1, \pm 2, \pm 4, \pm 8, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{4}{3}, \pm\frac{8}{3}$

Using the factor theorem, we can test each one of these until $f(a) = 0$. For this quadratic, $f(-4) = 0, f\left(\frac{2}{3}\right) = 0$

$$\begin{aligned} 3x^2 + 10x - 8 &= (x+4)\left[(3)\left(x - \tfrac{2}{3}\right)\right] \\ &= (x+4)(3x-2) \end{aligned}$$

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Ex.2 Determine all possible rational roots for

$f(x) = 2x^3 + 7x^2 - 64x - 105$

then show that $2x + 3$ is a factor.

(a) 105, factors : 1, 105, 3, 35, 5, 21, 7, 15 7×5 7×3 3×5

2, factors : 1, 2

all possible rational roots : $\dfrac{\pm 1, 3, 5, 7, 15, 21, 35, 105}{1, 2}$

list : $\pm 1, 3, 5, 7, 15, 21, 35, 105, \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7}{2}, \frac{15}{2}, \frac{21}{2}, \frac{35}{2}, \frac{105}{2}$

(b) $2x + 3 = 2\left(x + \frac{3}{2}\right)$

$f\left(\frac{-3}{2}\right) = 2\left(\frac{-3}{2}\right)^3 + 7\left(\frac{-3}{2}\right)^2 - \cancel{64}^{32}\left(\frac{-3}{\cancel{2}_1}\right) - 105$

$= \cancel{2}^{1}\left(\frac{-27}{\cancel{8}_4}\right) + 7\left(\frac{9}{4}\right) + 96 - 105$

$= \frac{-27}{4} + \frac{63}{4} - 9$ $\frac{4}{4} \times \frac{9}{1} = \frac{36}{4}$

$= \dfrac{36 - 36}{4}$

$f\left(\frac{-3}{2}\right) = 0$ $\therefore (2x+3)$ is a factor

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Assigned Work:

p.176 # [1 - 3][basics],
4bf, 5ac, 6abc, 7df, 8, 9, 10, 13, 17

$$\frac{D}{d} = Q + \frac{R}{d}$$

$$D = dQ + R$$

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10. $f(x) = ax^3 - x^2 + 2x + b$

$f(x) \div (x-1) \rightarrow R\ 10$

$f(1) = 10$ $\quad$ $f(2) = 51$

$a - 1 + 2 + b = 10$ $\quad$ $8a - 4 + 4 + b = 51$

$a + b = 9$ ① $\quad$ $8a + b = 51$ ②

remainder theorem:

$f(x) \div (x - a)$

$f(a) = R$

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13. $f(x) = x^3 + 4x^2 + kx - 4$

$f(-2) = R_1$ $\quad$ $f(2) = R_2$

$R_1 = 2R_2$

$-8 + 16 - 2k - 4 = R_1$ $\quad$ $8 + 16 + 2k - 4 = R_2$

$-2k + 4 = R_1$ $\quad$ $2k + 20 = R_2$

$-2k + 4 = 2R_2$ $\quad$ $2k + 20 = R_2$

$-2k + 4 = 2(2k + 20)$

$-2k + 4 = 4k + 40$

$-36 = 6k$

$k = -6$

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