Pascal's Triangle

Oct 3/2018

Discovered by Blaise Pascal in the 17th century, he discovered many patterns, including some related to combinations and probability.

1	Row 0	$t_{0,0}$
1 1	Row 1	$t_{1,0}$ $t_{1,1}$
1 2 1	Row 2	$t_{2,0}$ $t_{2,1}$ $t_{2,2}$
1 3 3 1	Row 3	$t_{3,0}$ $t_{3,1}$ $t_{3,2}$ $t_{3,3}$
1 4 6 4 1	Row 4	$t_{4,0}$ $t_{4,1}$ $t_{4,2}$ $t_{4,3}$ $t_{4,4}$

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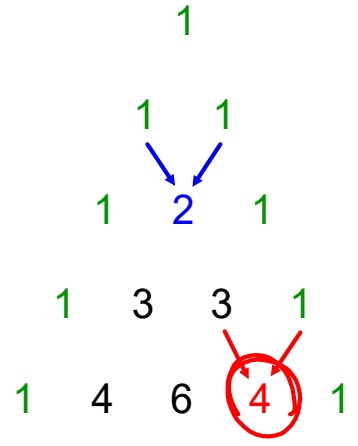
Pascal's method defines any term of the triangle as the sum of the two diagonal terms above it.

$$t_{n,r} + t_{n,r+1} = t_{n+1,r+1}$$

In practice, build the triangle starting at the top using an iterative process, one row at a time. Note the outer diagonals are always one.

$t_{4,3} = t_{3,2} + t_{3,3}$



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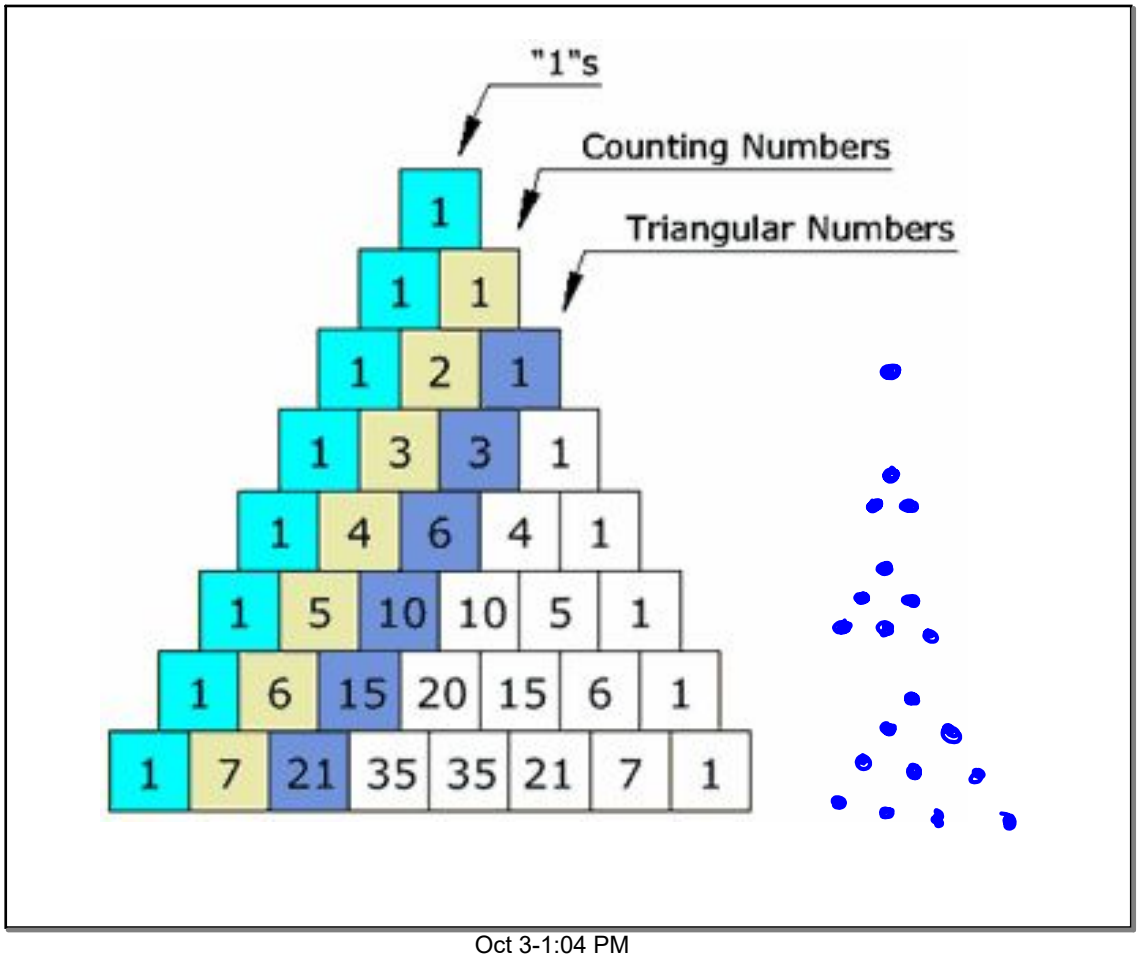

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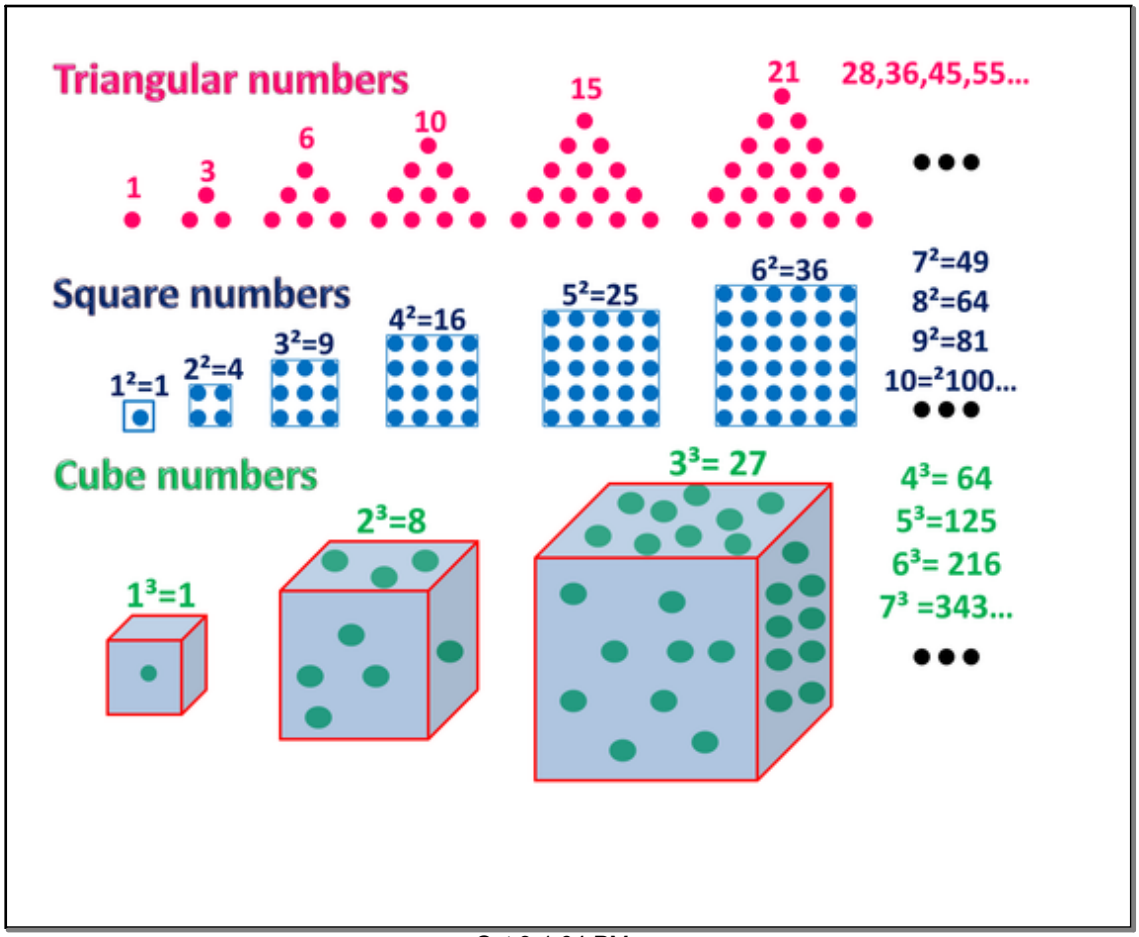

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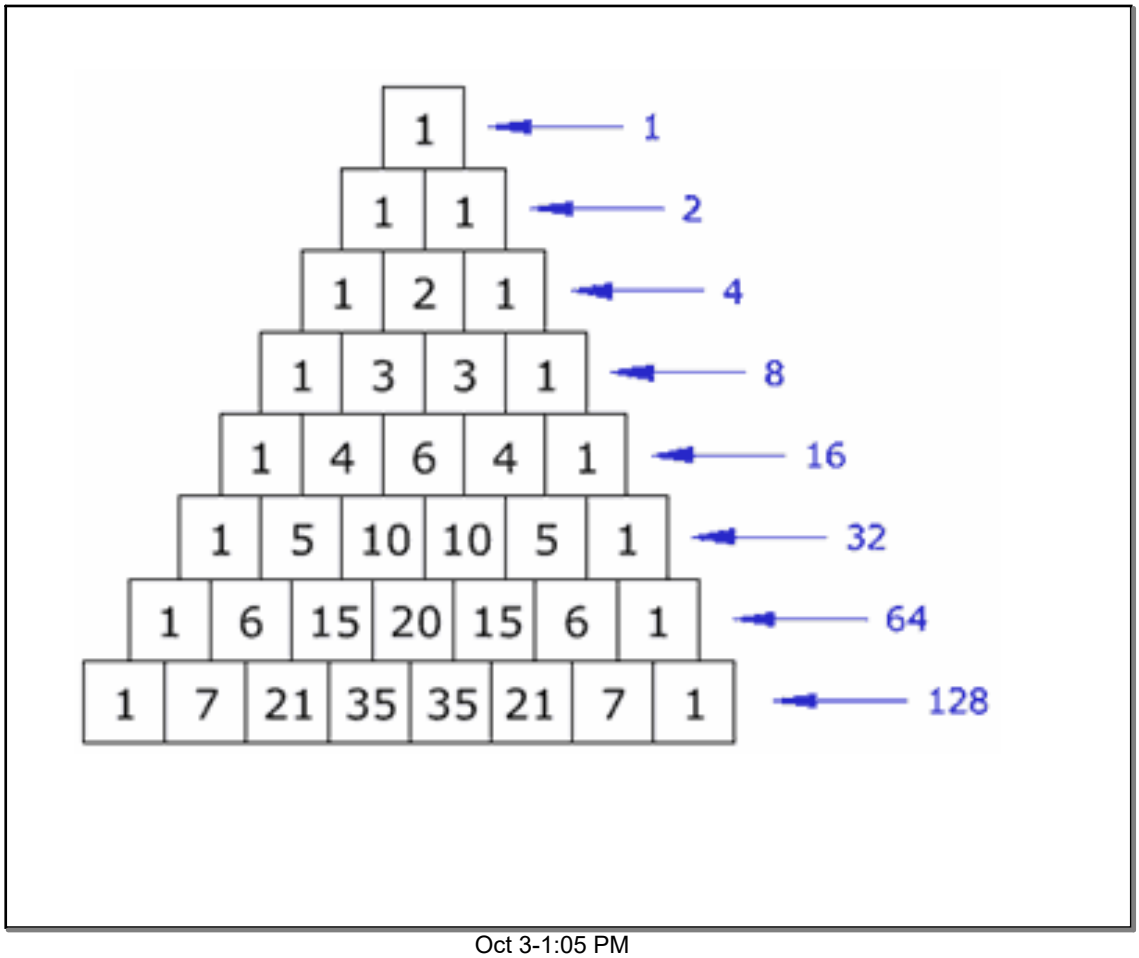

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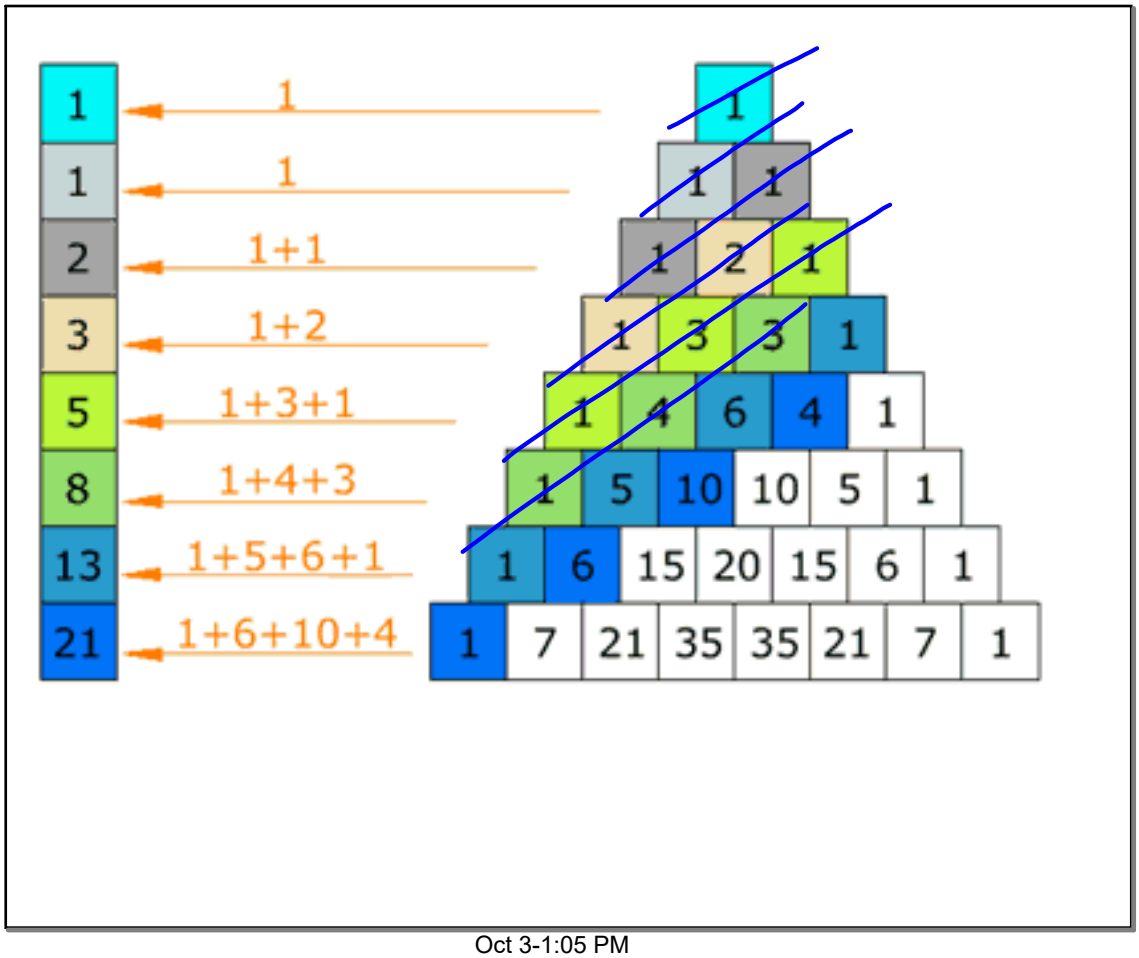

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The elements of each row can also be related to combinations.

1	Row 0	$_0C_0$
1 1	Row 1	$_1C_0$ $_1C_1$
1 2 1	Row 2	$_2C_0$ $_2C_1$ $_2C_2$
1 3 3 1	Row 3	$_3C_0$ $_3C_1$ $_3C_2$ $_3C_3$
1 4 6 4 1	Row 4	$_4C_0$ $_4C_1$ $_4C_2$ $_4C_3$ $_4C_4$

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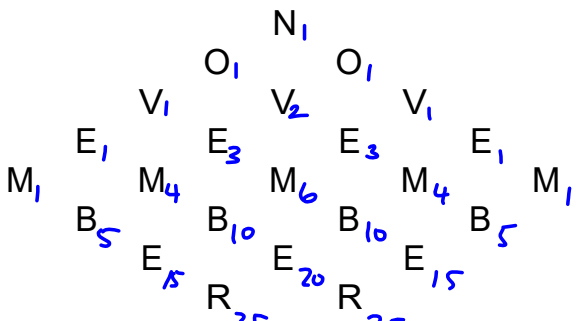
Ex. Use Pascal's method to determine how many paths spell "NOVEMBER", starting at the top and always moving down to the left or right.



two endings of R, 35 paths each

∴ there are 70 ways to spell November

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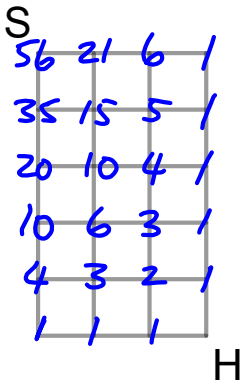
Ex. School is 5 blocks north and 3 west from home. How many paths from home to school?

Note: We have done a similar problem using combinations. Assume all movement is towards school.



∴ there are 56 paths directly from home to school.

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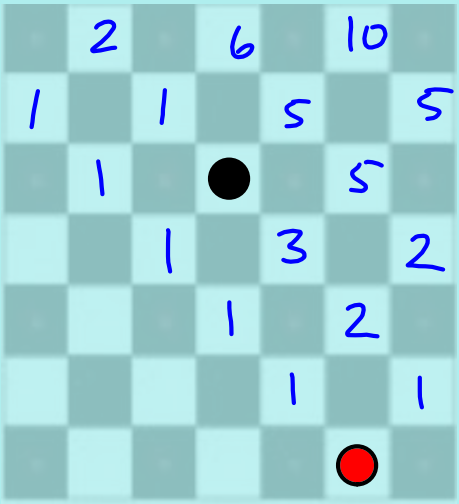
Ex. The red checker can only travel diagonally upward. How many paths to the top without crossing the black piece?



total = 18 ways

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Assigned Work:

p.126 # 1, 2, 4, 8, 9, 10, 11

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