Combinations

Oct 1/2018

Permutation - an arrangement of identifiable elements where order matters.

$$_nP_r = P(n,r) = \frac{n!}{(n-r)!}$$

"n permutations of r elements"

Combination - an arrangement of elements where order does not matter.

$$_nC_r = C(n,r) = \binom{n}{r} = \frac{n!}{(n-r)!r!}$$

"n combinations of r elements"
or
"n choose r"

removes effect of ordering r elements

Sep 30-10:47 AM

Ex. Five students (A, B, C, D, E) are running for student's council to fill positions of president, VP, and secretary.
(a) How many election results are possible?

$\underset{5}{\underline{P}} \times \underset{4}{\underline{VP}} \times \underset{3}{\underline{S}}$ or $_5P_3 = \frac{5!}{(5-3)!}$

$= 60$

* P, VP, S are uniquely identified, so use permutation.

(b) How many ways could these 5 students form a 3-member committee?

- being picked for committee, it does not matter 1st, 2nd, 3rd

→ order does not matter.

$$_5C_3 = \frac{5!}{(5-3)!\,3!}$$

$= 10$

Sep 30-11:26 AM

Ex. How many ways are there to deal a 5-card hand?

$$ {}_{52}C_5 = \frac{52!}{47!\,5!} $$

52 cards to choose from

groups of 5 cards

*order does not matter

$$ = 2\,598\,960 $$

Sep 30-11:03 AM

Ex. How many ways are there to deal a 5-card hand?

Assume the cards dealt are: As, Kc, Qh, Jd, 10s.

The order these cards are received does not matter.

<u>Identical hands</u>

As, Kc, Qh, Jd, 10s
As, Qh, Kc, Jd, 10s
As, Kc, Qh, 10s, Jd
10s, As, Kc, Qh, Jd
Kc, Qh, Jd, 10s, As
10s, Jd, Qh, Kc, As
...

$$ \frac{{}_{52}P_5}{5!} = \frac{\frac{52!}{(52-5)!}}{5!} $$

$$ {}_{52}C_5 = \frac{52!}{(52-5)!\,5!} $$

Sep 30-11:03 AM

Assigned Work:

p.113 # 2 - 5
8, 9, 12, 15, 17, 18

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9. From a standard deck, how many five-card hands contain the following?

a) only black cards

b) all face cards

c) no hearts

d) two red and three black cards

e) one face card

(a) ${}_{26}C_5$ OR $\frac{26(25)(24)(23)(22)}{5!}$

$= \frac{{}_{26}P_5}{5!}$

(c) ${}_{39}C_5$ (d) ${}_{26}C_2 \times {}_{26}C_3$

Oct 2-1:57 PM

15. Communication In a drama class of 18 students, nine are selected to be actors in a play, five will build sets, and four will be stage hands. In how many ways could the class be divided up?

a) Make your calculations by selecting the actors first.

b) Make your calculations by selecting the set builders first.

c) Compare your answers. Explain the results.

(a) $_{18}C_9 \times {}_9C_5 \times {}_4C_4$

$= 6126120$

(b) $_{18}C_5 \times {}_{13}C_9 \times {}_4C_4$

$= 612\,6120$

$_{18}C_9 \times {}_9C_5 \times {}_4C_4$

$= \frac{18!}{\cancel{9!}\,9!} \times \frac{\cancel{9!}}{4!\,5!} \times \frac{\cancel{4!}}{0!\,\cancel{4!}}$

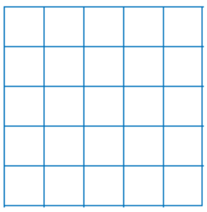
$_{18}C_5 \times {}_{13}C_9 \times {}_4C_4$

$= \frac{18!}{\cancel{13!}\,5!} \times \frac{\cancel{13!}}{4!\,9!} \times \frac{4!}{0!\,4!}$

$\frac{n!}{p!\,q!\,r!\ldots}$

Oct 2-1:57 PM

17. Ten identical playing pieces are placed on a 5 by 5 game board.



(a) $_{25}C_{10}$

\# locations

\# pieces

a) In how many ways could 10 playing pieces be placed on the board if there are no restrictions?

b) In how many ways could 10 playing pieces be placed on the board if there must be two pieces in each row?

c) Describe how the results would change if the playing pieces were all different.

(b) $_5C_2 \times {}_5C_2 \times {}_5C_2 \times {}_5C_2 \times {}_5C_2$

$= ({}_5C_2)^5$

(c) permutations, order matters

Oct 2-1:58 PM