Unit 3: Probability Distributions for Discrete Variables

Probability Distributions & Expected Value Oct 15/2018

Vocabulary:

(1) Discrete Random Variable: A variable that can have only certain values within a given range, with finite steps between values. For example, the sum of two dice.

(2) Continuous Random Variable. A variable that can have an infinite number of possible values within a given range. Often measurements such as volume, temperature, or time.

Note: The random variable is often called the independent variable in other branches of math.

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Ex. Probability Histogram

Tara is hoping to purchase a condo in a new building in Ottawa. The table shows the percentage of units being constructed with various numbers of rooms.

Number of Rooms, x	Percent, P(x)
1	15%
2	30%
3	35%
4	20%

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Ex. Probability Histogram

(a) Identify the random variable. x

(b) Construct a probability histogram.

(c) Calculate the sum of probabilities for each room type.

Number of Rooms, x	Percent, P(x)
1	15%
2	30%
3	35%
4	20%
	100%

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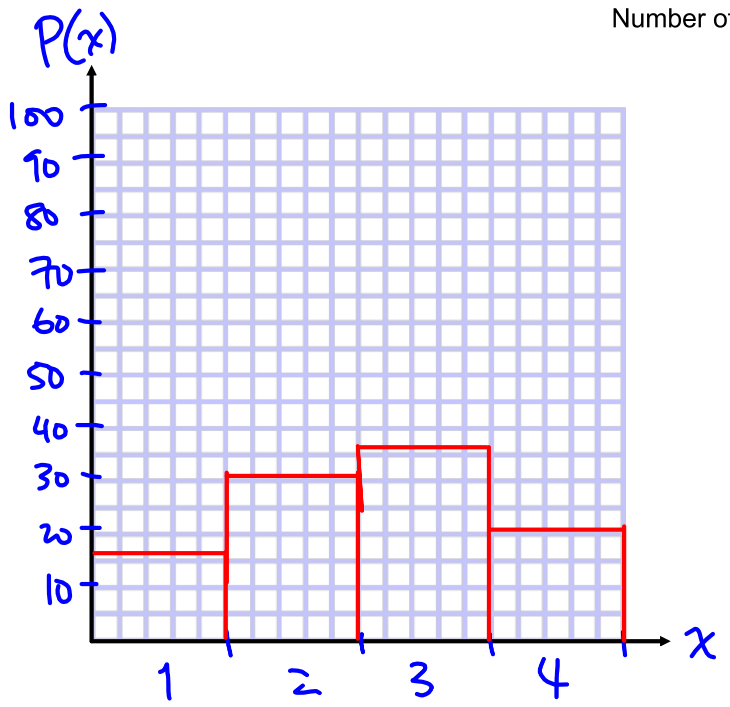
Ex. Probability Histogram

(b) Construct a probability histogram.



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Ex. Probability Histogram
(b) Construct a probability histogram.

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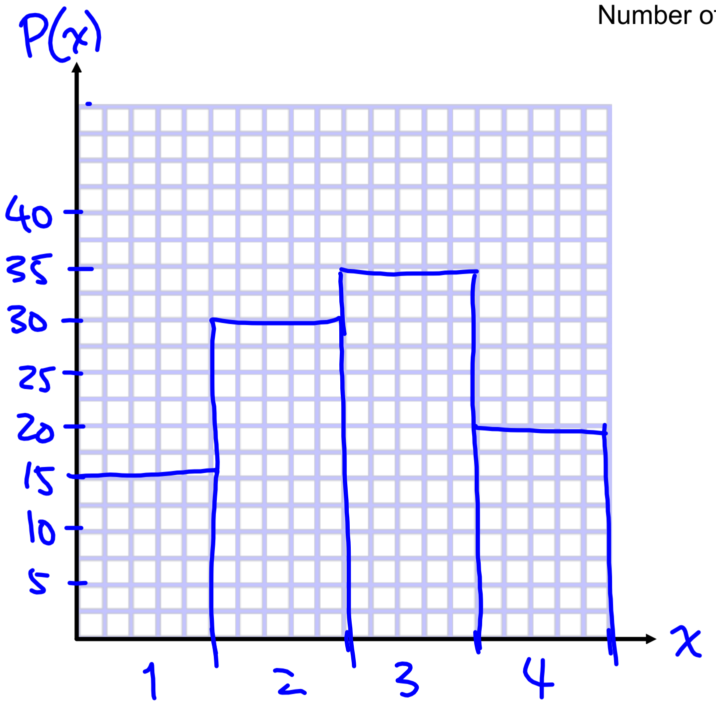

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Ex. Probability Histogram
(b) Construct a probability histogram.

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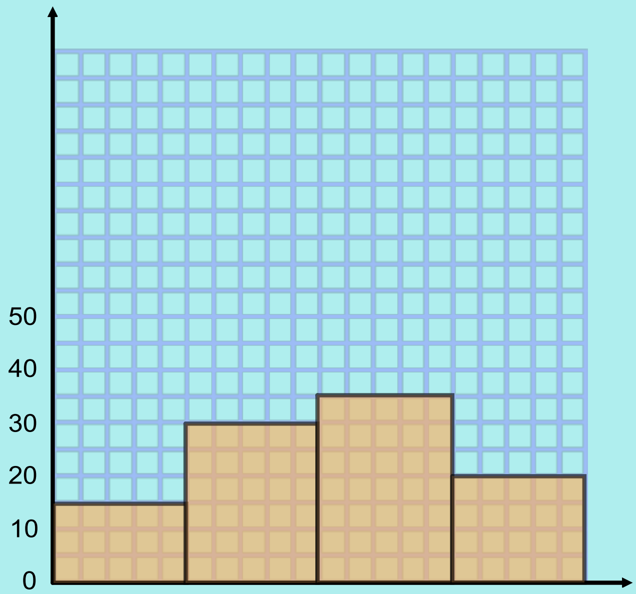

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Ex. Probability Histogram
(c) Calculate the sum of probabilities for each room type.

Number of Rooms, x	Percent, P(x)
1	15%
2	30%
3	35%
4	20%

"sigma" → sum.

$$\sum_{i=1}^{4} P(i) = P(1) + P(2) + P(3) + P(4)$$
$$= 0.15 + 0.30 + 0.35 + 0.20$$
$$= 1$$

"take sum of P(i), where i starts at 1 and ends at 4"

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Ex. If Tara were to select a condo at random, how many rooms would she <u>expect</u> to find?

Number of Rooms, x	Percent, P(x)
1	15%
2	30%
3	35%
4	20%

Try to figure this out on your own or with a partner!

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Ex. If Tara were to select a condo at random, how many rooms would she <u>expect</u> to find?

Number of Rooms, x	Percent, P(x)
1	15%
2	30%
3	35%
4	20%

Expected Value:

$$E(X) = \sum_{i=1}^{4} x_i P(i)$$

uppercase X → all x combined.

lowercase x → random variable

$$= x_1 P(1) + x_2 P(2) + x_3 P(3) + x_4 (P_4)$$
$$= (1)(0.15) + (2)(0.30) + (3)(0.35) + (4)(0.20)$$
$$= 2.6$$

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Number of Rooms, x	Percent, P(x)
1	15%
2	30%
3	35%
4	20%

Expected Value:

$$\begin{aligned} E(X) &= \sum_{i=1}^{4} x_i P(i) \\ &= 1 \cdot P(1) + 2 \cdot P(2) + 3 \cdot P(3) + 4 \cdot P(4) \\ &= 0.15 + 2(0.30) + 3(0.35) + 4(0.20) \\ &= 2.6 \end{aligned}$$

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Assigned Work:

p.151 # 1, 4, 5, 6, 9, 13, 17*

c d

1. Classify each of the random variables as discrete or continuous:
   a) the number of points scored in a basketball game
   b) the length of time players played in a basketball game
   c) the mass of the weights in a weight room
   d) the number of windows in the classrooms in a school
   e) the area of the windows in the classrooms in a school

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9. Thinking A school is holding a fundraising raffle. The first prize is $500, the three second prizes are $100 each, and the five third prizes are $50 each. A total of 2000 tickets were sold at $5 each.
   a) What is the probability of winning a prize?
   b) What is the expected payout per ticket?
   c) What is the expected profit per ticket?
   d) What price should have been charged to have a 90% profit per ticket?

prize (value)	# of winners	probability	E(x)
$500	1	$\frac{1}{2000}$	$500\left(\frac{1}{2000}\right)=\frac{1}{4}$
$100	3	$\frac{3}{2000}$	$100\left(\frac{3}{2000}\right)=\frac{3}{20}$
$50	5	$\frac{5}{2000}$	$50\left(\frac{5}{2000}\right)=\frac{1}{8}$
$0	1991	$\frac{1991}{2000}$	0
	2000		

(a) $P(\text{prize}) = \frac{n(\text{prize})}{n(\text{all})}$

$= \frac{9}{2000}$

(b) $E(X) = \sum_{i=1}^{3} x_i P(i)$

$= x_1 P(1) + x_2 P(2) + x_3 P(3)$

$= 500\left(\frac{1}{2000}\right) + 100\left(\frac{3}{2000}\right) + 50\left(\frac{5}{2000}\right)$

$= 0.525$

(c) profit = revenue - cost

$= 2000(5) - [1(500) + 3(100) + 5(50)]$

$= 8950$

profit per ticket $= \frac{8950}{2000}$

$= 4.475$

$E(x) + \text{profit per T} = 5 = \text{price}$

(d) 90% profit = 0.9 × price

$E(X) + 0.9\,\text{price} = \text{price}$

$0.525 = 0.1\,\text{price}$

$\frac{0.525}{0.1} = \text{price}$

price = 5.25

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17. What is the expected sum of two weighted dice on which the number 5 occurs twice as often as the other numbers?

roll	prob
1	$a = \frac{1}{7}$
2	$a = \frac{1}{7}$
3	$a = \frac{1}{7}$
4	$a = \frac{1}{7}$
5	$2a = \frac{2}{7}$
6	$a = \frac{1}{7}$

$7a = 1$

$a = \frac{1}{7}$

$E(X_1) = \frac{1}{7}(1+2+3+4+5+5+6)$

$= \frac{26}{7}$ for a single die

$E(X_2) = \frac{26}{7}$

$E(X) = E(X_1) + E(X_2)$

$= \frac{52}{7}$

① treat as a 7-sided die, with '5' on two sides.

	1	2	3	4	5	5	6
1	2	3	4	5	6	6	7
2	3	4	5	6	7	7	8
3	4	5	6	7	8	8	9
4	5	6	7	8	9	9	10
5	6	7	8	9	10	10	11
5	6	7	8	9	10	10	11
6	7	8	9	10	11	11	12

x	2	3	4	5	6	7	8	9	10	11	12
freq	1	2	3	4	7	8	7	6	6	4	1
prob	$\frac{1}{49}$	$\frac{2}{49}$									

$$E(X) = \sum_{i=1}^{11} x_i P(i)$$

$$= 2\left(\frac{1}{49}\right) + 3\left(\frac{2}{49}\right) + \cdots$$

$$= \frac{2(1) + 3(2) + 4(3) + 5(4) + \cdots}{49}$$

$$= \frac{364}{49}$$

$\doteq$

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