Binomial Distributions   Oct 18/2018

A binomial distribution has a specified number of independent trials where there are only two possible outcomes, success or failure. The probability of a success is the same in each trial.
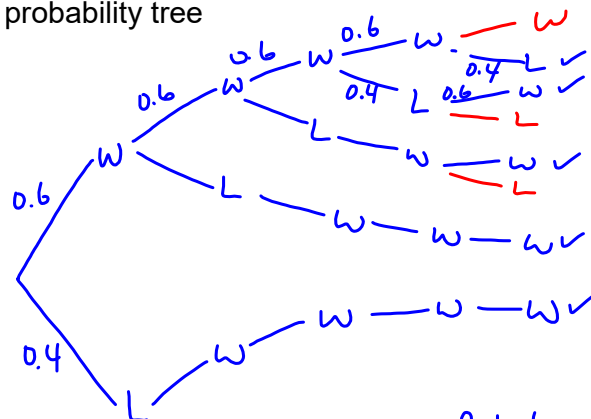
Recall: Pascal's triangle was connected to the Binomial Theorem, used to expand $(p + q)^n$.

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Oct 16-9:14 PM

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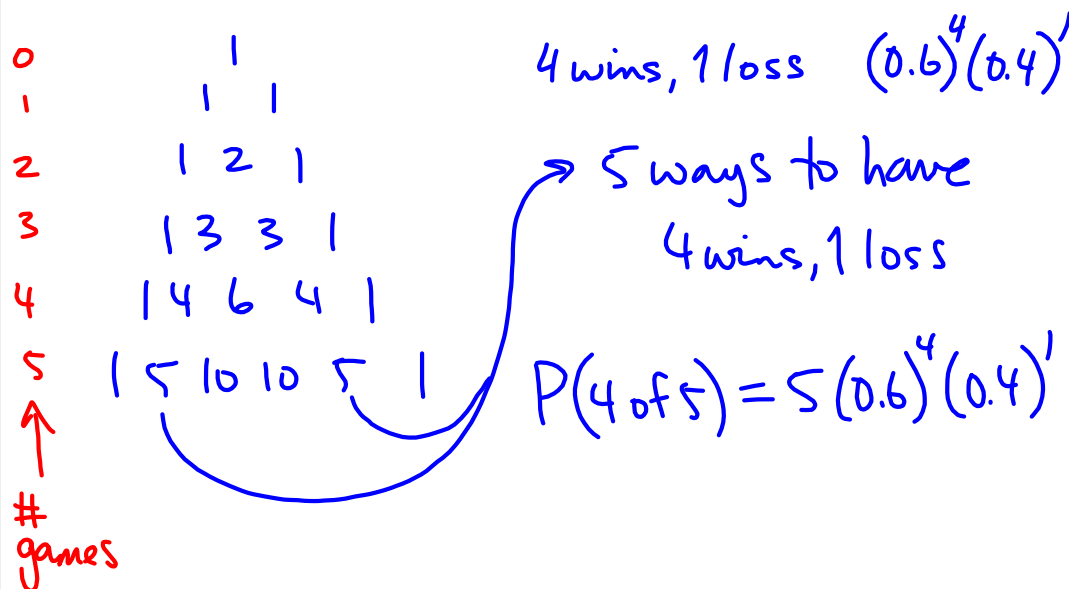
(1) probability tree

5 successful branches

each branch prob = $(0.6)^4 (0.4)$

$$P(4 \text{ of } 5) = 5 (0.6)^4 (0.4)$$
$$= 0.2592$$
$$= 25.92\%$$

Oct 16-9:14 PM

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(2) Pascal's triangle

$$0 \qquad 1$$
$$1 \qquad 1 \quad 1$$
$$2 \qquad 1 \quad 2 \quad 1$$
$$3 \qquad 1 \quad 3 \quad 3 \quad 1$$
$$4 \qquad 1 \quad 4 \quad 6 \quad 4 \quad 1$$
$$5 \qquad 1 \quad 5 \quad 10 \quad 10 \quad 5 \quad 1$$

games

4 wins, 1 loss $(0.6)^4 (0.4)^1$

$\Rightarrow$ 5 ways to have 4 wins, 1 loss

$$P(4 \text{ of } 5) = 5 (0.6)^4 (0.4)^1$$

Oct 16-9:14 PM

The probability of x successes in n identical <u>independent</u> trials is:

$$P(x) = {}_nC_x p^x q^{n-x}$$

n is the number of trials
x is the number of successes
p is the probability of success in a single trial
q is the probability of failure in a single trial

These are the same terms as the expansion of $(p + q)^n$.

Note: q = 1 - p, or p + q = 1.

Oct 16-9:29 PM

Ex. During the regular season, the Ottawa Senators have a 55% chance of winning any individual game. What is the probability of winning <u>exactly</u> 5 of the next 10 games?

$$P(x) = {}_nC_x \, p^x q^{n-x}$$

$P = 0.55$

$q = 1 - P$
$\quad = 0.45$

$n = 10$

$x = 5$

$$P(5) = {}_{10}C_5 \, (0.55)^5 (0.45)^{10-5}$$

$$\doteq 0.2340$$

$$\doteq 23\%$$

Oct 16-9:44 PM

Expected Value in a Binomial Distribution

Recall:　$$E(X) = \sum_{i=1}^{n} x_i P(i)$$

Oct 16-9:33 PM

Expected Value in a Binomial Distribution

$$E(x) = np$$

where:　n is the number of trials
　　　　p is the probability of success for one trial

Ex. A card is drawn at random from a regular deck and then replaced.
(a) In 20 trials, what is the probability of 3 aces?
(b) In 20 trials, how many aces are expected?

(a)　$P_{ace} = \frac{1}{13}$　　　$P(3 \text{ aces}) = {}_{20}C_3 \left(\frac{1}{13}\right)^3 \left(\frac{12}{13}\right)^{17}$

　　$n = 20$
　　$x = 3$　　　　　　　$\doteq 13.3\%$
　　$q = \frac{12}{13}$　　　　　$\doteq 0.133$

(b)　$E(x) = np$
　　　　$= 20\left(\frac{1}{13}\right)$　　$\therefore$ expect 1.54
　　　　$\doteq 1.54$　　　　　　aces in
　　　　　　　　　　　　　　20 draws.

Oct 16-9:33 PM

Assigned Work:

p.167 # 1, 2, 4, 5a (by hand), 5b (tech), 7,
　　　8, 9, 12, 13, 16, 17

Feel free to use a spreadsheet for any questions

Oct 18-8:50 AM

5. Prepare a probability table and a graph for a binomial distribution with

   a) $n = 6$ and $p = 0.3$
   b) $n = 8$ and $p = \frac{1}{9}$

prob. of 'success'
per event $\quad p = 0.3$
$q = 0.7$

# of events

# Successes	prob
x	$_nC_x(p)^x(q)^{n-x}$
0	$_6C_0(0.3)^0(0.7)^6 =$
1	$_6C_1(0.3)^1(0.7)^5 =$
2	
3	
4	
5	
6	$_6C_6(0.3)^6(0.7)^0 =$

prob

success

0 1 2 3 4 5 6

Oct 22-1:57 PM

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8. Six people are asked to choose a number between 1 and 20. What is the probability that

   a) two people choose the number 9?
   b) at least two people choose the number 9?

let $A =$ a person chooses 9

$p = \frac{1}{20} \quad q = \frac{19}{20} \quad n = 6$

(a) $P(2) = {_6C_2}\left(\frac{1}{20}\right)^2\left(\frac{19}{20}\right)^4$

(b) $P(\geq 2) = \underbrace{P(2) + P(3) + P(4) + P(5) + P(6)}_{\text{direct method}}$

OR

$P(<2) = P(0) + P(1)$

$P(\geq 2) = 1 - P(<2)$

$\quad = 1 - \Big(P(0) + P(1)\Big)$

$\quad = 1 - {_6C_0}\left(\frac{1}{20}\right)^0\left(\frac{19}{20}\right)^6 - {_6C_1}\left(\frac{1}{20}\right)^1\left(\frac{19}{20}\right)^5$

Oct 22-1:57 PM

17. Opinion polls based on small samples often yield misleading results. In a particular city, 65% of residents are opposed to a new light rail transit system.

    a) If a poll were taken, calculate the probabilities of a majority of people approving the transit system with a sample of $> 50\%$
       - 7 people
       - 100 people
       - 1000 people

    b) Explain any differences in the results.

Majority

(a) 7 people   4

     100   51

     1000   501

$$E(X) = np$$

$$7 \quad\quad 7(0.65) = 4.55$$
$$\doteq 5$$

$$100 \quad\quad 100(0.65) = 65$$

$$1000 \quad\quad 1000(0.65) = 650$$

Oct 22-1:57 PM