

Unit 5: Trigonometric Identities & Equations

Equivalent Trigonometric Functions

Nov. 14/2018

Due to the periodic nature of trigonometric functions, there are multiple (infinite) ways to express equivalent functions.

(1) Using the period:

Both sine and cosine have a period of 2π , which means any phase shift by a multiple of the period will be equivalent.

$$\sin(\theta) = \sin(\theta + 2\pi) = \sin(\theta - 2\pi)$$

$$\cos(\theta) = \cos(\theta + 2\pi) = \cos(\theta - 2\pi)$$

Similarly, for tangent,

$$\tan(\theta) = \tan(\theta + \pi) = \tan(\theta - \pi)$$

Nov 6-7:25 PM

(2) By symmetry:

Recall, even functions: $f(x) = f(-x)$

odd functions: $f(-x) = -f(x)$

or
 $f(x) = -f(-x)$

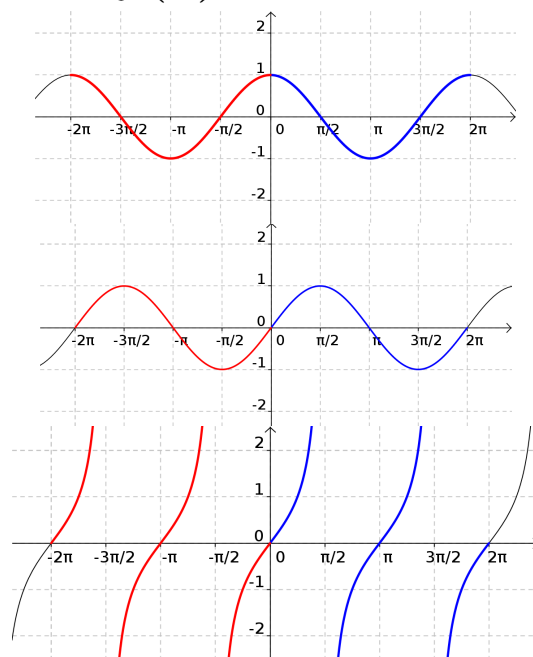
Cosine is even (reflective symmetry across the y-axis)

$$\cos(x) = \cos(-x)$$

Sine and tangent are odd (rotational symmetry)

$$\sin(-x) = -\sin(x)$$

$$\tan(-x) = -\tan(x)$$

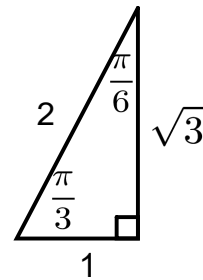


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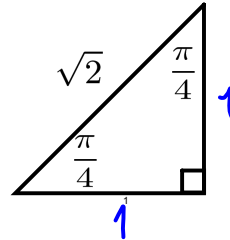
(3) Using complimentary angles:

Recall: Complimentary angles add to $\frac{\pi}{2}$ (or 90°)

$$\sin\left(\frac{\pi}{3}\right) = \cos\frac{\pi}{6} \quad \csc\left(\frac{\pi}{3}\right) = \sec\frac{\pi}{6}$$



$$\cos\left(\frac{\pi}{3}\right) = \sin\frac{\pi}{6} \quad \sec\left(\frac{\pi}{3}\right) = \csc\frac{\pi}{6}$$



$$\tan\left(\frac{\pi}{3}\right) = \cot\frac{\pi}{6} \quad \cot\left(\frac{\pi}{3}\right) = \tan\frac{\pi}{6}$$

$$\sin(\theta) = \cos\left(\frac{\pi}{2} - \theta\right) = \cos\left(-\left(\theta - \frac{\pi}{2}\right)\right)$$

$$\cos(\theta) = \sin\left(\frac{\pi}{2} - \theta\right)$$

Nov 6-7:50 PM

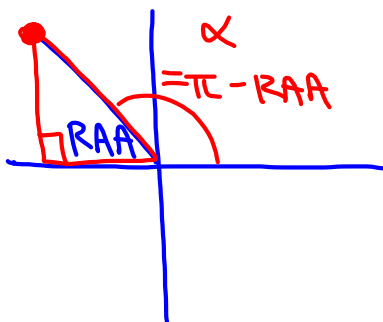
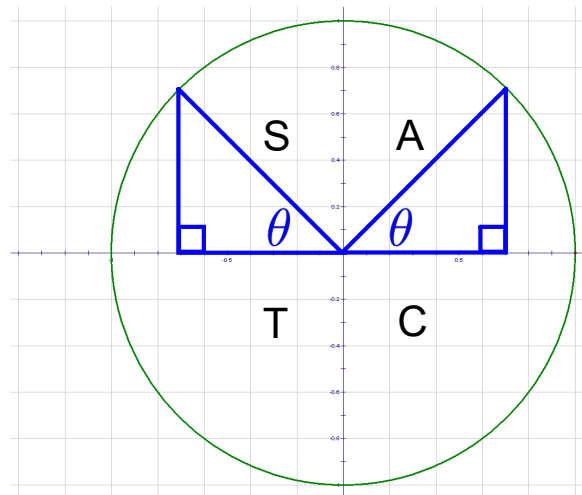
(4) Using primary and related acute angles:

The trigonometric ratio for an angle in any quadrant can be expressed using the RAA and the CAST rule.

$$\sin(\pi - \theta) = +\sin\theta$$

$$\cos(\pi - \theta) = -\cos\theta$$

$$\tan(\pi - \theta) = -\tan\theta$$

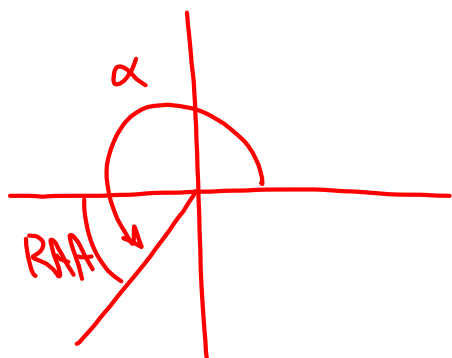
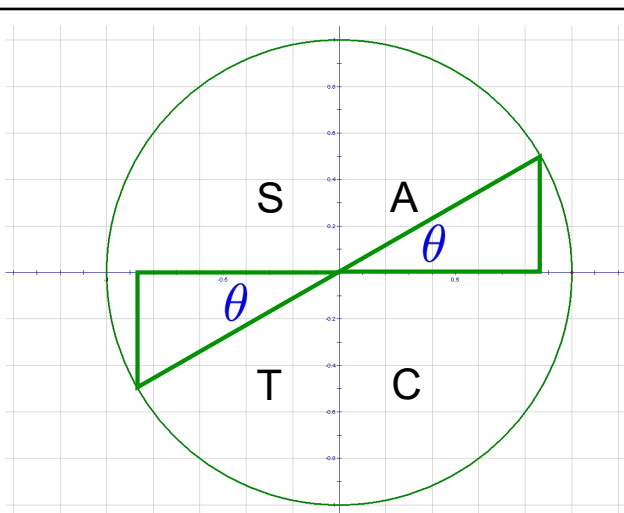


Nov 6-8:02 PM

$$\sin(\pi + \theta) = -\sin \theta$$

$$\cos(\pi + \theta) = -\cos \theta$$

$$\tan(\pi + \theta) = +\tan \theta$$

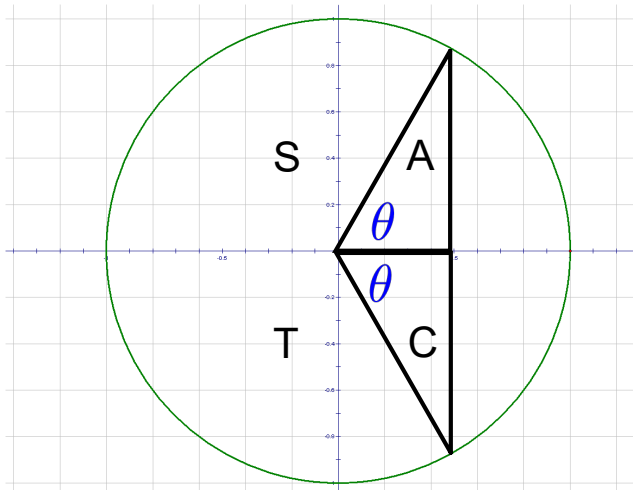


Nov 6-8:12 PM

$$\sin(2\pi - \theta) = -\sin \theta$$

$$\cos(2\pi - \theta) = +\cos \theta$$

$$\tan(2\pi - \theta) = -\tan \theta$$

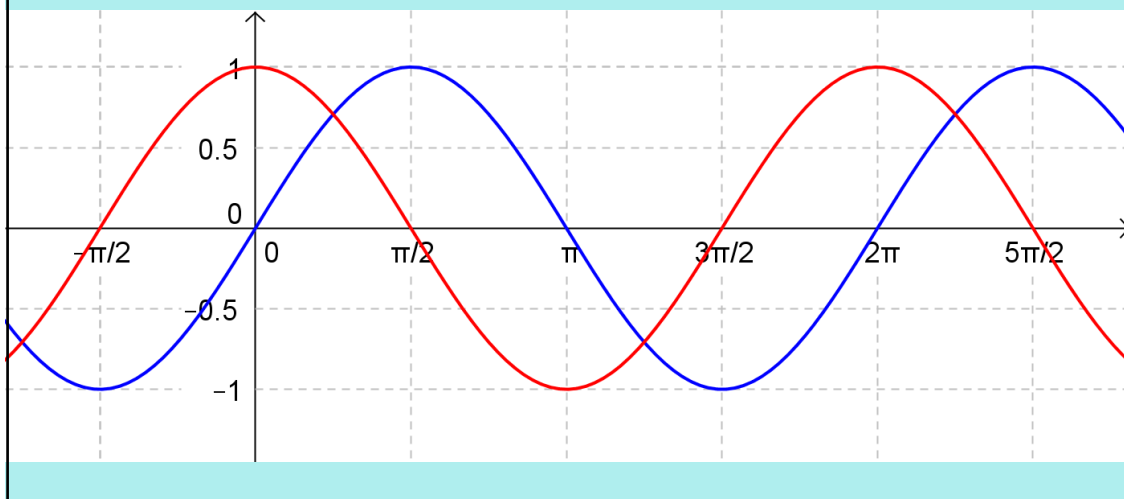


Nov 6-8:12 PM

(5) By transformations (reflections & phase shift):

Due to the periodic nature of all graphs, and how certain pairs are so similar (sine/cosine, tangent/cotangent, secant/cosecant), it is possible to verify equivalent expressions from the graphs through an application of transformations.

Ex.1 Verify



Nov 6-8:20 PM

Assigned Work:

p.392 # 1, 2a, 3acd, 4a, 5abc

Nov 6-9:56 PM