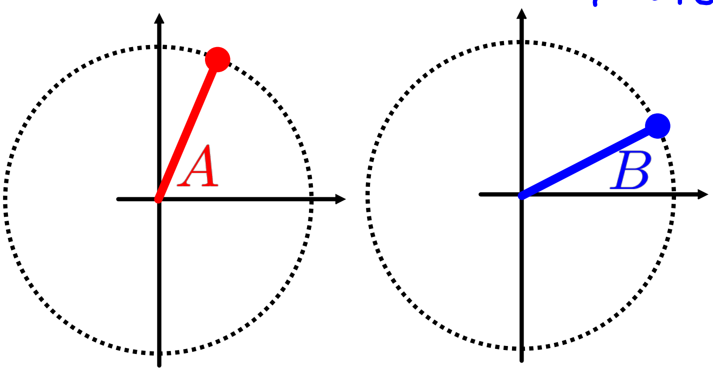
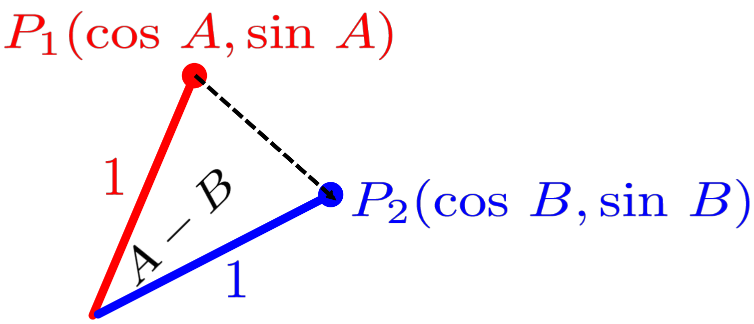
Compound Angle Formulas

Nov. 15/2018

Consider the angles 'A' and 'B' on the unit circle.





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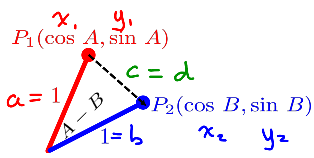
Cosine Law: $c^2 = a^2 + b^2 - 2ab\cos C$

Distance Formula: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Use cosine law and distance formula to develop an expression in terms of 'A' and 'B':



$$c = d$$
$$c^2 = d^2$$

$$a^2 + b^2 - 2ab\cos C = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

$$1^2 + 1^2 - 2(1)(1)\cos(A-B) = (\cos B - \cos A)^2 + (\sin B - \sin A)^2$$

$$2 - 2\cos(A-B) = \cos^2 B - 2\cos B\cos A + \cos^2 A$$
$$+ \sin^2 B - 2\sin B\sin A + \sin^2 A$$

$\boxed{\sin^2 B = (\sin B)^2}$

$$\frac{\cancel{2} - 2\cos(A-B)}{-2} = \frac{\cancel{2} - 2\cos A\cos B}{-2} \frac{- 2\sin A\sin B}{-2}$$

$$\cos(A-B) = \cos A\cos B + \sin A\sin B$$

$$f(A-B) = f(A)f(B) + g(A)g(B)$$

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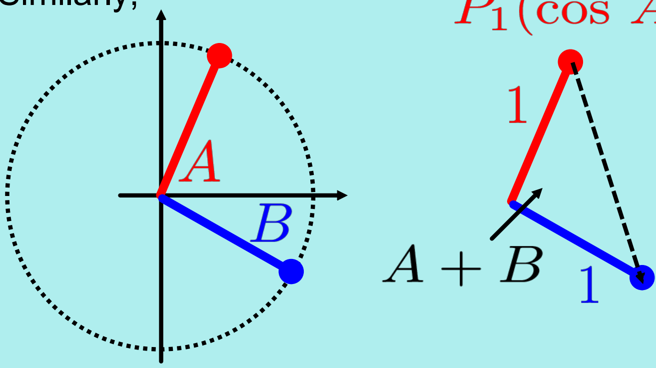
Similarly,



$P_1(\cos\ A, \sin\ A)$

$P_2(\cos A, \sin(-B))$

$= P_2(\cos A, -\sin B)$

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

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Similar compound angle formulas can be obtained for sine using the complimentary angle formula:

$$\sin\theta = \cos\left(\frac{\pi}{2} - \theta\right)$$

Ex.1 $\sin(A+B) = \cos\left(\frac{\pi}{2} - (A+B)\right)$

$= \cos\left(\frac{\pi}{2} - A - B\right)$

$= \cos\left(\left(\frac{\pi}{2} - A\right) - B\right)$

$= \cos(\theta - B)$

$= \cos\theta \cos B + \sin\theta \sin B$

$= \cos\left(\frac{\pi}{2} - A\right)\cos B + \sin\left(\frac{\pi}{2} - A\right)\sin B$

$\sin(A+B) = \sin A \cos B + \cos A \sin B$

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For the tangent function, use the quotient identity:

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

Ex.2 $\tan(A+B)$

$$= \frac{\sin(A+B)}{\cos(A+B)}$$

$$= \frac{\sin A\cos B + \cos A\sin B}{\cos A\cos B - \sin A\sin B}$$

$$= \frac{\cos A\cos B\left(\frac{\sin A\cos B}{\cos A\cos B} + \frac{\cos A\sin B}{\cos A\cos B}\right)}{\cos A\cos B\left(1 - \frac{\sin A\sin B}{\cos A\cos B}\right)}$$

$$= \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

$$\boxed{\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}}$$

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Many applications of the compound angle formulas involve angles from the special triangles.

Ex.1 Simplify and then evaluate:

$$\cos\frac{7\pi}{12}\cos\frac{5\pi}{12} + \sin\frac{7\pi}{12}\sin\frac{5\pi}{12}$$

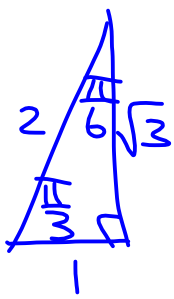
recall: $\cos A\cos B + \sin A\sin B = \cos(A-B)$

$$= \cos\left(\frac{7\pi}{12} - \frac{5\pi}{12}\right)$$

$$= \cos\left(\frac{2\pi}{12}\right)$$

$$= \cos\left(\frac{\pi}{6}\right)$$

$$= \frac{\sqrt{3}}{2}$$



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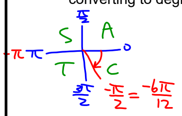
Ex.2 Determine an exact value for $\tan\left(\frac{-5\pi}{12}\right)$

notes:
(1) simplest to convert to RAA and apply CAST
(2) easier to see sum or difference of special angles by converting to degrees, then back to radians



$$\tan\left(-\frac{5\pi}{12}\right) = -\tan\left(\frac{5\pi}{12}\right)$$
$$= -\tan(75^\circ)$$
$$= -\tan(30^\circ + 45^\circ)$$
$$= -\tan\left(\frac{\pi}{6} + \frac{\pi}{4}\right)$$
$$= -\left[\frac{\tan\frac{\pi}{6} + \tan\frac{\pi}{4}}{1 - \tan\frac{\pi}{6}\tan\frac{\pi}{4}}\right]$$
$$= -\left[\frac{\frac{\sqrt{3}}{3} + 1}{1 - \frac{\sqrt{3}}{3}(1)}\right]$$
$$= -\left[\frac{\frac{\sqrt{3}+3}{3}}{\frac{3-\sqrt{3}}{3}}\right]$$
$$= -\left[\frac{\sqrt{3}+3}{3} \times \frac{3}{3-\sqrt{3}}\right]$$
$$= -\left[\frac{\sqrt{3}+3}{3-\sqrt{3}}\right] \times \frac{3+\sqrt{3}}{3+\sqrt{3}}$$
$$= -\left[\frac{3\sqrt{3} + (\sqrt{3})^2 + 9 + 3\sqrt{3}}{9 - (\sqrt{3})^2}\right]$$

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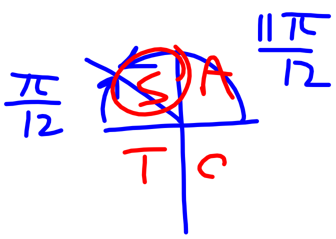
Assigned Work:

p.400 # 1-4, 5acf, 6cde, 8, 9ade, 10, 13
c

$$8(c)\quad \cos\frac{11\pi}{12} = -\cos\frac{\pi}{12}$$

① lesson #1



$$= -\cos 15^\circ$$
$$= -\cos(45^\circ - 30^\circ)$$
$$= -\cos\left(\frac{\pi}{4} - \frac{\pi}{6}\right)$$
$$= -\left[\cos\frac{\pi}{4}\cos\frac{\pi}{6} + \sin\frac{\pi}{4}\sin\frac{\pi}{6}\right]$$

Nov 6-9:56 PM

9(a) $\sin\theta = \frac{4}{5}$

$0 < \theta < \frac{\pi}{2}$

Q1

$\sin\theta = \frac{y}{r}$

$\frac{4}{5} = \frac{y_\theta}{r_\theta}$

$x^2 + y^2 = r^2$

$x^2 + 4^2 = 5^2$

$x^2 = 9$

$x_\theta = \pm 3$

$x = 3$, Q1

$\sin\alpha = \frac{-12}{13}$

$\frac{3\pi}{2} < \alpha < 2\pi$

Q4

$\sin\alpha = \frac{y_\alpha}{r_\alpha}$

$\frac{-12}{13} = \frac{y}{r}$

$y = -12 \quad r = 13$

$x = \pm 5$

$x_\alpha = 5$, Q4

$$\cos(\theta+\alpha) = \cos\theta\cos\alpha - \sin\theta\sin\alpha$$

$$= \left(\frac{3}{5}\right)\left(\frac{5}{13}\right) - \left(\frac{4}{5}\right)\left(\frac{-12}{13}\right)$$

Nov 16-10:43 AM

13. $$\frac{\sin(f+g) + \sin(f-g)}{\cos(f+g) + \cos(f-g)}$$

$$= \frac{\sin f\cos g + \cancel{\cos f\sin g} + \sin f\cos g - \cancel{\cos f\sin g}}{\cos f\cos g - \cancel{\sin f\sin g} + \cos f\cos g + \cancel{\sin f\sin g}}$$

$$= \frac{\cancel{2}\sin f\cancel{\cos g}}{\cancel{2}\cos f\cancel{\cos g}}$$

$$= \tan f, \quad \cos g \neq 0$$

infinite number of holes

at $x = \frac{\pi}{2} + n\pi, n \in \mathbb{Z}$

Nov 16-10:53 AM