Proving Trigonometric Identities

Nov. 19/2018

An identity is an equation which is always true for all values of the variable(s) within the domain.

To prove a trigonometric identity, manipulate one side of the equality until a form identical to the other side is reached.

When using an identity to solve for a variable, any restrictions where information is lost (e.g., dividing out factors) must be noted and incorporated into your final answer(s).

It is also possible to disprove an equality through a counterexample. If the equality can be shown false with a single example, it is considered to be false in general.

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Tips for working with trig identities:

 1. Keep your goal in mind! As you work one side, keep
        in mind how to get closer to your target.

 2. Start with the most complicated side and try to
        make it simpler.

 3. If stuck, try expressing in terms of sine and cosine.

 4. Only work on one side at a time. Only switch sides
        if you cannot progress any further (e.g., you are
        stuck, or you will make it more complicated)

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Ex.1 Prove $\dfrac{\cos(x-y)}{\cos(x+y)} = \dfrac{1 + \tan x\ \tan y}{1 - \tan x\ \tan y}$

$$LS = \frac{\cos(x-y)}{\cos(x+y)}$$

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$= \frac{\cos x \cos y + \sin x \sin y}{\cos x \cos y - \sin x \sin y}$$

$$x^2 + x$$
$$= x\left(\frac{x^2 + x}{x}\right)$$

$$= \frac{\left(\frac{\cos x \cos y}{\cos x \cos y} + \frac{\sin x \sin y}{\cos x \cos y}\right)\left(\cos x \cos y\right)}{\left(\frac{\cos x \cos y - \sin x \sin y}{\cos x \cos y}\right)\left(\cos x \cos y\right)}$$

$$= \frac{1 + \tan x \tan y}{1 - \tan x \tan y}$$

$$= RS \checkmark$$

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Some educators prefer a one-sided proof, where only one side of the equality is turned into the other.

This view has merit, as it maintains an important property of a proof like this, called <u>reversibility</u>.

Ex.1 Prove $\dfrac{\cos(x-y)}{\cos(x+y)} = \dfrac{1 + \tan x\ \tan y}{1 - \tan x\ \tan y}$

$LS = \sim$ ①　　　　　$RS = \sim$ ①

$\quad = \sim\sim\sim$ ②　　　　$= \sim\sim\sim$ ②

$\quad = \sim\sim\sim$ ③　　　　$= \underline{\sim\sim}$ ③

$\quad = \underline{\sim\sim}$ ④　　　$LS = RS$

$LS = $ ①
$\quad = $ ②
$\quad = $ ③
$\quad = $ ④ $= $ ③
$\quad = $ ②
$\quad = $ ①
$\quad = RS$

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Assigned Work:

p.416 # 5, 8, (9, 10, 11)(odd), (17)

b e
e

10 (b) $LS = \sin^2\theta + \cos^4\theta$ $RS = \cos^2\theta + \sin^4\theta$

$LS = \left(1 - \cos^2\theta\right) + \left(\cos^2\theta\right)^2$

$= 1 - \cos^2\theta + \left(1 - \sin^2\theta\right)^2$

$= 1 - \cos^2\theta + 1 - 2\sin^2\theta + \sin^4\theta$

$= 2 - \cos^2\theta - 2\left(1 - \cos^2\theta\right) + \sin^4\theta$

$= 2 - \cos^2\theta - 2 + 2\cos^2\theta + \sin^4\theta$

$= \cos^2\theta + \sin^4\theta$

$= RS \checkmark$

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10(e)

$LS = \sin\left(\frac{\pi}{4} + x\right) + \sin\left(\frac{\pi}{4} - x\right)$ $RS = \sqrt{2}\cos x$

$\sin(A+B) = \sin A \cos B + \cos A \sin B$

$= \sin\frac{\pi}{4}\cos x + \cos\frac{\pi}{4}\sin x + \sin\frac{\pi}{4}\cos x - \cos\frac{\pi}{4}\sin x$

$= 2\sin\frac{\pi}{4}\cos x$

$= 2\left(\frac{\sqrt{2}}{2}\right)\cos x$

$= \sqrt{2}\cos x$

$= RS$

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11(e) $LS = \cot \theta - \tan \theta$

$RS = 2 \cot 2\theta$

$= \dfrac{2}{\tan 2\theta}$

$= \dfrac{2}{\dfrac{2\tan \theta}{1 - \tan^2 \theta}}$

$= \dfrac{2(1 - \tan^2 \theta)}{2\tan \theta}$

$= \dfrac{1}{\tan \theta} - \dfrac{\tan^2 \theta}{\tan \theta}$

$= \cot \theta - \tan \theta$

$= LS \checkmark$

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17.

$8\cos^4 x = a\cos 4x + b\cos 2x + c$

$\cos 2\theta = 2\cos^2 \theta - 1$ ①

$\cos 4\theta = 2\cos^2 2\theta - 1$ ②

$LS = 8\cos^4 x$ ①: $2\cos^2 x = \cos 2x + 1$

$= 8(\cos^2 x)^2$

$= 2(2\cos^2 x)^2$

$= 2(\cos 2x + 1)^2$

$= 2(\cos^2 2x + 2\cos 2x + 1)$

$= 2\cos^2(2x) + 4\cos(2x) + 2$

$= (\cos 4x + 1) + 4\cos 2x + 2$

$= \underbrace{1}_{a}\cos 4x + \underbrace{4}_{b}\cos 2x + \underbrace{3}_{c}$

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