Confidence Intervals

Dec. 4/2018

Example: A recent poll (sample) shows that a political party's support is $34\% \pm 3\%$, 19 times out of 20.

Definitions:

(1) margin of error: A statement of how much variation you can expect for a particular measurement (3%).

(2) confidence interval: The range of possible values for a measurement and margin of error (31% to 37%).

(3) confidence level: The probability that a statistic is within the specified confidence interval (19/20, or 95%).

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To calculate the margin of error:

$$E = z\sqrt{\frac{p(1-p)}{n}}$$

E is the margin of error
p is the probability based on statistical data
z is the z-Score for the required confidence level
n is the size of the sample

Confidence Level	z-Score
90%	1.645
95%	1.96
99%	2.576

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Ex. A political survey polled 1000 eligible voters, and 40% supported the incumbent candidate. What is the margin of error for a 95% confidence level?

$$E = z\sqrt{\frac{p(1-p)}{n}}$$

$$= 1.96\sqrt{\frac{0.4(1-0.4)}{1000}}$$

$$\doteq 0.03036$$

$\therefore$ incumbent has a 40% ± 3.0% chance of winning, $\underbrace{\text{19 times out of 20}}_{95\%}$.

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<u>Repeated sampling</u> occurs when you have a population with <u>known</u> statistics which follows a normal distribution.

The sample mean will also be normally distributed, and the sample data will have a standard deviation:

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n_s}}$$

standard deviation of population.

The margin of error for the sample mean will be:

$$E = z\frac{\sigma}{\sqrt{n_s}} = z\sigma_{\bar{x}}$$

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Ex. At a town fair, giant pumpkins were entered in a contest, with (sample) masses (in kg):

11, 13, 15, 18, 12, 14, 10, 16

Past (population) results suggest a mean of 14.2 kg with a standard deviation of 2.5 kg.

Determine a 90% confidence interval for the sample mean.

$$\sigma_s = \frac{\sigma_p}{\sqrt{n}} = \frac{2.5}{\sqrt{8}} \doteq 0.884$$

$$E = z_{90}\sigma_s = (1.645)(0.884)$$
$$E \doteq 1.454$$

$$\bar{x} = \frac{\sum x_i}{8} = 13.625$$

$\therefore$ pumkins will have masses 13.6 kg ± 1.45 kg, with 90% confidence

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Assigned Work:

p.359 # 4, 5, 7, 9, 11, 14

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7. Application A Single Crème cookie is made using a cream filling between two wafers. The amount of cream follows a normal distribution with a mean of 25 g and a standard deviation of 2.0 g. The company claims its new Double Crème line contains twice the amount of filling. A random sample of 20 such cookies were found to contain cream content as shown.

Mass of Cream (g)				
48.9	47.3	47.3	45.5	52.9
50.1	46.0	47.9	48.5	48.2
47.5	51.9	49.7	47.8	50.1
46.9	51.0	45.9	45.4	47.1

a) Calculate the mean of the sample and the standard deviation for the sample means. What assumption must you make?

b) Determine the 95% confidence interval for the sample mean.

c) Is the company justified in claiming that the Double Crème line contains twice the filling of the Single Crème line? Give reasons for your answer.

$\mu = 25\,g$

$\sigma_p = 2.0\,g$

$n_s = 20$

(a) $\bar{x} = \dfrac{\sum x_i}{n_s}$

$= \underline{\qquad\qquad}$

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