Evaluating Logarithms

Dec 5/2018

$$\boxed{y = \log_a x \text{ is equivalent to } x = a^y}$$

For many problems, we can obtain an exact value by switching between these equivalent expressions and looking for a common base.

There are also some general rules we can develop.

Ex.1 Solve

(a) $y = log_3 3^2$

$y = \log_a x$

$x = a^y$

$3^2 = 3^y$

$\Rightarrow y = 2$

(b) $y = log_4 4^7$

$4^7 = 4^y$

$\Rightarrow y = 7$

Nov 27-7:50 PM

In general: $\boxed{log_a a^x = x}$ (1)

Ex.2 Evaluate:

(a) $log_{10}(1)$

$= \log_{10} 10^0$

$= 0$

(b) $log_5(1)$

$= \log_5 5^0$

$= 0$

Dec 1-7:19 PM

In general: $\boxed{log_a 1 = 0}$ (2)

Ex.3 Evaluate:

(a) $2^{\underbrace{log_2 x}}$

$f(x) = 2^x$ $f^{-1}(x) = log_2 x$

$f(f^{-1}(x)) = x$

$2^{log_2 x} = x$

(b) $5^{log_5 x} = y$

$b^z = y$

$z = log_b y$

$log_5 x = log_5 y$

$\Rightarrow x = y$

$5^{log_5 x} = x$

Dec 1-7:22 PM

In general: $\boxed{a^{log_a x} = x}$ (3)

What if no common base is possible, and these general rules cannot be applied?

Recall: Many calculators only allow for a base of 10 or 'e'.

$y = \log_{10} x$ or $y = \ln x$

For different bases, we can still calculate the value of a logarithm by using an equivalent expression.

$$\boxed{\log_a x = \frac{\log_{10} x}{\log_{10} a}}$$ (4)

Note: we will derive this in our lesson on "laws of logarithms"

Dec 1-7:26 PM

Assigned Work:

p.466 # 3 (using rule 1 or 2)
5 (using 1 or 2)
8 (using 4)
6, 9, 12, 17

12. $m(t) = P\left(\frac{1}{2}\right)^{\frac{t}{h}}$ $h = 1620$

(a) $P = 5g$ $t = 150$

(b) $M(t) = 4g$, $t = ?$

$\underbrace{4 = 5\left(\frac{1}{2}\right)^{\frac{t}{1620}}}_{y = k(a^x)}$

$y = a^x$

$x = \log_a y$

$\frac{4}{5} = \left(\frac{1}{2}\right)^{\frac{t}{1620}}$

$y = a^x$

$x = \log_a y$

$\frac{t}{1620} = \log_{\frac{1}{2}}\left(\frac{4}{5}\right)$

$\log_a x = \frac{\log_{10} x}{\log_{10} a}$

$t = 1620 \log_{\frac{1}{2}}\left(\frac{4}{5}\right)$

$t = 1620\left(\frac{\log_{10}\left(\frac{4}{5}\right)}{\log_{10}\left(\frac{1}{2}\right)}\right)$

$t \doteq$ _____

Nov 27-8:36 PM