

Laws of Logarithms

Recall: Exponent Laws

same base

$$(a^x)(a^y) = a^{x+y}$$

$$a^x \div a^y = \frac{a^x}{a^y} = a^{x-y}, \quad a \neq 0$$

$$a^{-x} = \frac{1}{a^x}, \quad a \neq 0$$

$$(a^x)^y = a^{xy}$$

$$a^0 = 1, \quad a \neq 0$$

different bases

$$(ab)^x = (a^x)(b^x)$$

$$\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}, \quad b \neq 0$$

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Ex. Derive the Product Law for Logarithms
(use the product law for exponents)

$$(a^x)(a^y) = a^{x+y}$$

P q r

$$P = a^x \Rightarrow x = \log_a P$$

$$q = a^y \Rightarrow y = \log_a q$$

$$r = a^{x+y} \Rightarrow x+y = \log_a r$$

$$\log_a P + \log_a q = \log_a r$$

$$\boxed{\log_a P + \log_a q = \log_a Pq}$$

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Ex. Derive the Quotient Law for Logarithms
(use the quotient law for exponents) $\frac{a^x}{a^y} = a^{x-y}$

$$\frac{P}{Q} = r$$

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Ex. Derive the Power Law for Logarithms
(use the power law for exponents) $(a^x)^y = a^{xy}$

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Laws of Logarithms

Dec 7/2018

product law: $\log_a xy = \log_a x + \log_a y$

quotient law: $\log_a \left(\frac{x}{y} \right) = \log_a x - \log_a y$

power law: $\log_a x^r = r \log_a x$

Ex.1 Simplify then evaluate (no log calculations!):

(a) $\log_3 6 + \log_3 4.5$

(b) $\log_2 48 - \log_2 3$

(c) $\log_5 \sqrt[3]{25}$

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Ex.1 Simplify then evaluate (no log calculations!):

(a) $\log_3 6 + \log_3 4.5$

(b) $\log_2 48 - \log_2 3$

(c) $\log_5 \sqrt[3]{25}$

$$\log_a xy = \log_a x + \log_a y$$

$$\log_a \left(\frac{x}{y} \right) = \log_a x - \log_a y$$

$$\log_a x^r = r \log_a x$$

(a) $\log_3 6 + \log_3 4.5$ (b) $\log_2 48 - \log_2 3$
 $= \log_3 [(6)(4.5)]$ $= \log_2 \left(\frac{48}{3} \right)$
 $= \log_3 27$ $= \log_2 16$
 $= \underline{\log_3 3^3}$ $= \log_2 2^4$
 $= 3$ $= 4$

(c) $\log_5 \sqrt[3]{25}$
 $= \log_5 25^{\frac{1}{3}}$
 $= \frac{1}{3} \log_5 25$
 $= \frac{1}{3} \log_5 5^2$
 $= \frac{1}{3}(2)$
 $= \frac{2}{3}$

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Ex.2 Use the power law to show $\log_a x = \frac{\log_{10} x}{\log_{10} a}$

$$\log_a x^r = r \log_a x$$

let $x = a^y$

$y = \log_a x$

$$\log_{10} x = \log_{10} a^y$$

$$\log_{10} x = y \log_{10} a$$

$$\frac{\log_{10} x}{\log_{10} a} = y$$

$$\frac{\log_{10} x}{\log_{10} a} = \log_a x$$

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Ex.3 Rewrite as a single log to a common base:

$$\log 12 + \frac{1}{2} \log 7 - \log 2$$

$$= \log 12 + \log 7^{\frac{1}{2}} - \log 2$$

BEDMAS

$$\log_a xy = \log_a x + \log_a y$$

$$\log_a \left(\frac{x}{y} \right) = \log_a x - \log_a y$$

$$\log_a x^r = r \log_a x$$

$$= \log [(12)(7^{\frac{1}{2}})] - \log 2$$

$$= \log \left[\frac{(12)(\sqrt{7})}{2} \right]$$

$$= \log (6\sqrt{7})$$

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Assigned Work:

p.475 # 5 (look past obvious answer!),
 # 6, 7, 9ace, 10ace, 11bde, 17
 a

$$\begin{aligned}
 b(a) & \log_{25} 5^3 \\
 & = 3 \log_{25} 5 \\
 & = 3 \log_{25} (25)^{\frac{1}{2}} \\
 & = 3 \left(\frac{1}{2}\right) \\
 & = \frac{3}{2}
 \end{aligned}$$

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II (d)

$$\begin{aligned}
 & \log_2 x^2 - \log_2 xy + \log_2 y^2 \\
 & = \log_2 \left(\frac{x^2}{xy} \right) + \log_2 y^2 \\
 & = \log_2 \left(\frac{x^2 y^2}{xy} \right) \\
 & = \log_2 xy
 \end{aligned}$$

= $\log_2 x^2 y^2 - \log_2 xy$
 = $\log_2 \frac{x^2 y^2}{xy}$

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9(a)

$$\log_4 3 + \frac{1}{2} \log_4 8 - \log_4 2$$

$$= \log_4 3 + \log_4 8^{\frac{1}{2}} - \log_4 2$$

$$= \log_4 \left[\frac{3\sqrt{8}}{2} \right]$$

$$= \log_4 \left[\frac{3 \cdot 2\sqrt{2}}{2} \right]$$

$$= \log_4 (3\sqrt{2})$$

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Attachments

In.class