Solving Exponential and Logarithmic Equations

The definition and properties of logarithms can be used to solve equations in which either powers or logarithms appear. If the unknown occurs in an exponent then the strategy is to isolate it by taking the logarithm of both sides.

Ex.1 Solve $3^{x+2} = 4$
(a) using definition of logarithms.
(b) by taking the log (base 10) of both sides.

check your solution!

(a) $y = a^x \Rightarrow x = \log_a y$

$4 = 3^{x+2} \qquad x+2 = \log_3 4$

$x = (\log_3 4) - 2$

$x = \left(\frac{\log 4}{\log 3}\right) - 2$

$x \doteq -0.738$

(b) $3^{x+2} = 4$

$\log 3^{x+2} = \log 4$

$(x+2)\log 3 = \log 4$

$x+2 = \frac{\log 4}{\log 3}$

$x = \left(\frac{\log 4}{\log 3}\right) - 2$

* only valid if all terms are positive, since $\log_a x$, $x > 0$

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Ex.2 Solve $\log_2 x - \log_2 3 = \log_2 6$

$\log_2 x = \log_2 6 + \log_2 3$

$\log_2 x = \log_2 18$

$\Rightarrow \quad x = 18$

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Ex.3 Solve $6^{3x} = 4^{2x-3}$

$$\log 6^{3x} = \log 4^{2x-3}$$

$$(3x)\log 6 = (2x-3)\log 4$$

$$3x\log 6 = 2x\log 4 - 3\log 4$$

$$3x\log 6 - 2x\log 4 = -3\log 4$$

$$x(3\log 6 - 2\log 4) = -3\log 4$$

$$x = \frac{-3\log 4}{3\log 6 - 2\log 4}$$

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Ex.4 Solve $\log_x 0.04 = -2$

$$y = \log_a x$$

$$x = a^y$$

$$x^{-2} = 0.04$$

$$\frac{1}{x^2} = 0.04$$

$$\frac{1}{0.04} = x^2$$

$$25 = x^2$$

$$x = \pm 5$$

but $x > 0$ (base of exp or log)

$$\therefore x = 5$$

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Ex.5 Solve $\log(x+2)+\log(x-1)=1$

check your solution!

$$\log_{10}[(x+2)(x-1)]=1$$

def'n force common base

$$10^1=(x+2)(x-1) \qquad \log[(x+2)(x-1)]=\log_{10}10$$

$$\Rightarrow (x+2)(x-1)=10$$

$$x^2+x-2=10$$
$$x^2+x-12=0$$
$$(x+4)(x-3)=0$$
$$x=-4,\ x=3$$

$y=\log_a x$

$a>0,\ x>0$

but base & argument of log positive

$$x+2>0 \qquad x-1>0$$
$$x>-2 \qquad x>1$$

$\therefore$ discard $x=-4$

$$LS=\log(3+2)+\log(3-1)$$
$$=\log 5+\log 2$$
$$=\log_{10}10$$
$$=1$$
$$=RS$$

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Assigned Work:

p.485 # 2, 8ad, 10, 17c
p.491 # 4, 5, 7, 12 — for next class

p.485 # 4, 6bc, 7, 11
p.492 # 3, 9 — work period

$$8(a)\quad 4^{x+1}+4^x=160$$
$$4^1(4^x)+4^x=160$$
$$4a+a=160$$
$$5a=160$$
$$a=32$$
$$4^x=32$$

$$x=\log_4 32$$
$$x=\_\_\_\_$$

$$(2^2)^x=2^5$$
$$2^{2x}=2^5$$
$$\Rightarrow 2x=5$$

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10.(d) $x = \log_2(53.2)$

$$= \frac{\log 53.2}{\log 2}$$

$$= \underline{\qquad\qquad}$$

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6 b c

$A = P(1+r)^n$ → # of comp. per.

rate = interest per comp. period

(b) $5000 = (1000)\left(1+\frac{0.12}{12}\right)^n$

$r = \frac{i}{t}$

interest per year $= \frac{0.12}{12}$

# of comp. per. per year

$5 = (1.01)^n$

$\Rightarrow \log_{1.01} 5 = n$

$n = \frac{\log 5}{\log 1.01}$

$n \doteq 161.747$

$\log(5) = \log(1.01^n)$

$\log 5 = n \log 1.01$

$\frac{\log 5}{\log 1.01} = n$

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17. (c) $3(2^x) = 4^{x+1}$

$$\log\left[3(2^x)\right] = \log\left[4^{x+1}\right]$$

$$\log 3 + \log 2^x = \log 4^{x+1}$$

$$\log 3 + x\log 2 = (x+1)\log 4$$

$$\log 3 + x\log 2 = x\log 4 + \log 4$$

$$x\log 2 - x\log 4 = \log 4 - \log 3$$

$$x(\log 2 - \log 4) = \log 4 - \log 3$$

$$x = \frac{\log 4 - \log 3}{\log 2 - \log 4}$$

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$$a = b$$

$$3a = 3b$$

$$a^2 = b^2$$

$$2^a = 2^b$$

$$\exp(a) = \exp(b)$$

$$* \; f(a) = f(b)$$

$2^x = \exp(x)$

$f(x) = \log(x)$

$$\log(a) = \log(b)$$

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