Solving Problems with Logarithmic Functions

Dec 12/2018

pH Scale (hydrogen ion concentration):

$$pH = -\log_{10} H^{+}$$

where pH is the scaled measurement (0 to 14)
and H+ is the concentration of hydrogen ions (mol/L)
(see p.494 for pH scale examples)

Ex. Calculate the pH for a hydrogen ion concentration of $\underbrace{0.00025 \text{ mol/L}}_{H^+}$. Is it an acid or base?

$pH = -\log(0.00025)$
$\doteq 3.6$

$\therefore$ it is an acid.

	pH
more basic	14
↑	
neutral	7
↓	
more acidic	0

Dec 7-5:58 PM

Richter Scale (earthquakes):

$$M = \log_{10} A$$

where M is the magnitude (approximately 0 to 10)
and A is the amplitude on the seismograph

Note: This formula is useful on for comparing the relative intensity of earthquakes. The actual energy of the earthquake is more complex.

Ex. How does an earthquake of magnitude 8 compare to an earthquake of magnitude 4.5?

$8 = \log_{10} A_8$ OR A_1

$4.5 = \log_{10} A_{4.5}$ OR A_2

$10^8 = A_8$

$10^{4.5} = A_{4.5}$

$\frac{A_8}{A_{4.5}} = \frac{10^8}{10^{4.5}}$
$= 10^{3.5}$
$\doteq 3162$

$\therefore$ the M8 is 3162 times the amplitude of the M4.5.

Dec 7-5:51 PM

Sound Loudness (decibel scale):

$$L = 10\log_{10}\left(\frac{I}{I_0}\right)$$

where L is the loudness of the sound,
I is the sound intensity (energy), and
I_0 is the threshold of human hearing

note: The threshold, I_0, is not always necessary for a useful calculation. $I_0 = 10^{-12}$ W/m^2

Ex. How does the sound intensity of a rock concert compare to that of a subway (see p.498 for loudness values)?

Want I_R vs I_S $L_R = 120$

$L_S = 90$

$120 = 10\log\frac{I_R}{I_0}$ $90 = 10\log\frac{I_S}{I_0}$

$12 = \log\frac{I_R}{I_0}$ $9 = \log\frac{I_S}{I_0}$

$10^{12} = \frac{I_R}{I_0}$ $I_S = 10^9 I_0$

$I_R = 10^{12} I_0$

$\frac{I_R}{I_S} = \frac{10^{12}\cancel{I_0}}{10^9\cancel{I_0}}$

$= 1000$

$\therefore$ the RC has 1000 times the energy of the subway

Dec 7-6:04 PM

Assigned Work:

p.499 # 1, 2, 3, 5a, 6a, 11, 13, 15

3. $I_0 = 10^{-12}$

$\frac{I}{I_0} = 10^6$

$L = 10\log\frac{I}{I_0}$

$L = 10\log_{10} 10^6$

$= 60$

$\therefore$ the sound is 60 dB

Dec 7-6:13 PM

11. $P(t) = P_0(a^t)$

sub point (0, 850)

$850 = P_0 a^0$

$850 = P_0$

$P(t) = 850\, a^t$

sub (7, 2250)

$2250 = 850\, a^7$

$\frac{2250}{850} = a^7$

$a = \left(\frac{2250}{850}\right)^{\frac{1}{7}}$

$a \doteq 1.149$

sub all other points to find a, take average a value as answer.

sub (42, 287200)

$287200 = 850\, a^{42}$

$a = \left(\frac{287200}{850}\right)^{\frac{1}{42}}$

$a \doteq 1.149$

Dec 13-9:18 AM

13. start 100%

cycle 1 lost 2.1% remain 97.9% $(0.979)(1)$

cycle 2 lost 2.1% of 97.9% remain 97.9% of 97.9% $(0.979)^2(1)$

3 $(0.979)^3$

100% = 1

$F(n) = (1)(0.979)^n$

amount of fluid

# of cycles

% of fluid remaining after each cycle

$0.5 = (1)(0.979)^n$

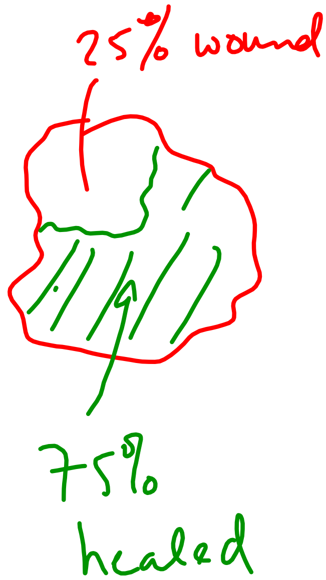
50% left

solve for n

Dec 13-9:24 AM

15. $A(t) = \underline{80}(10^{-0.023t})$

$\underbrace{0.25(80)}_{\text{25\% of original area}} = 80(10^{-0.023t})$



Dec 13-9:33 AM

WS #3

$w(t) = w_0(r)^t$ ← # of days of actual weight loss

$w(2) = 98$
$w(18) = 95$

OR

$w(t) = w_0(r)^{t-2}$ ← # of days since starting program

$\underbrace{w(4) = 98 \qquad w(20) = 95}_{\text{find } r}$

Dec 14-10:31 AM

WS#6.

from #4 : $T = (T_0 - S)e^{kt} + S$

Newton's law of cooling

does this apply to heating?

$T_0 = -10°C \quad S = 20°C$

$T = (-10 - 20)e^{kt} + 20$

$T = -30e^{kt} + 20$

at 10am, $T = -2°C$
$t = 3$ hours

$T(3) = -30e^{k(3)} + 20$

$-2 = -30e^{3k} + 20$

$\frac{-22}{-30} = e^{3k}$

use $\ln x$ as inverse

$\ln\left(\frac{22}{30}\right) = \ln\left(e^{3k}\right)$

$\ln\left(\frac{22}{30}\right) = 3k$

$k = \frac{1}{3}\ln\left(\frac{22}{30}\right)$

want T at noon, $T(5) = ?$

Dec 14-10:37 AM