

## Products of Functions

$$(f \times g)(x) = f(x) \times g(x)$$

"f times g of x"

Jan 7/2019

overlap

The domain of the combined function is the intersection of the domains of the original functions.

$$D_{f \times g} = D_f \cap D_g$$

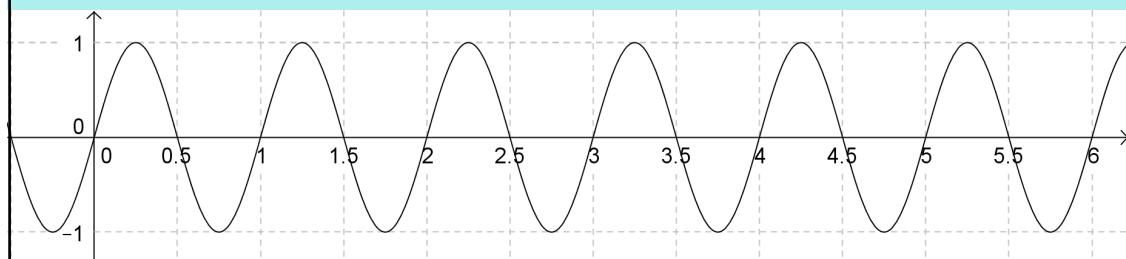
General Techniques:

- (1) Graphing: Multiply y-values for common x-values.
- (2) Algebra: Multiply expressions, simplify product, state restrictions (i.e., domain) based on intersection of original domains.

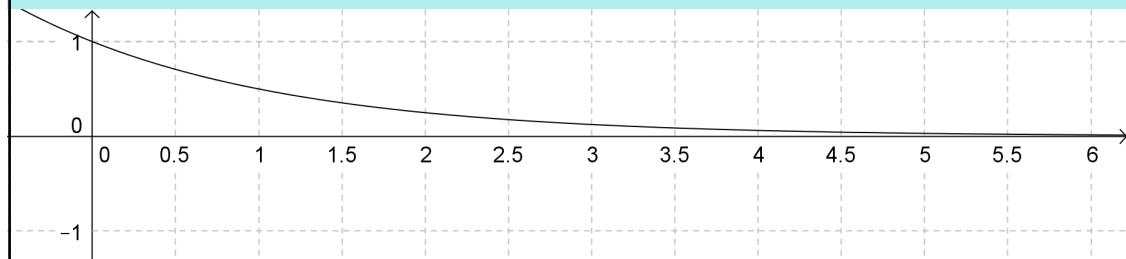
Jan 7-8:29 AM

## Application - Damped Harmonic Motion

A perfect spring will oscillate forever:



More realistically, the oscillation will decay over time:



Jan 7-8:30 AM

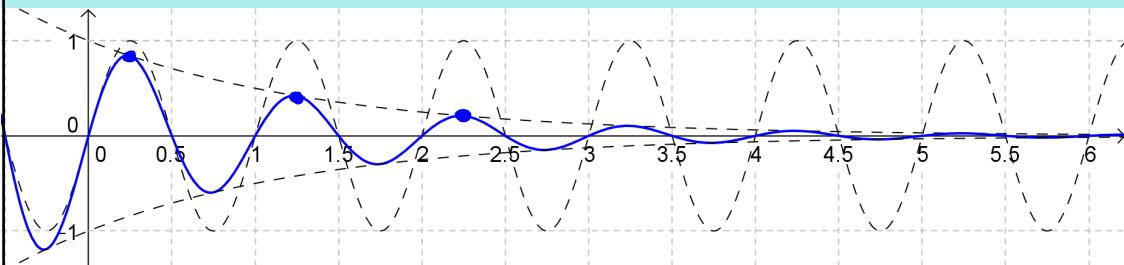
### Application - Damped Harmonic Motion

Periodic Function:  $f(x) = \sin(2\pi x)$

Exponential Decay:  $g(x) = 2^{-x}$

Damped Harmonic Function:

$$(f \times g)(x) = 2^{-x} \sin(2\pi x)$$



Jan 7-8:30 AM

Ex.1 Given  $f = \{(1,3), (2,-5), (3,7)\}$   
 $g = \{(2,-2), (3,3), (4,1)\}$

note: same general idea as graphing

determine  $f \times g$ .

$$D_f = \{1, 2, 3\}$$

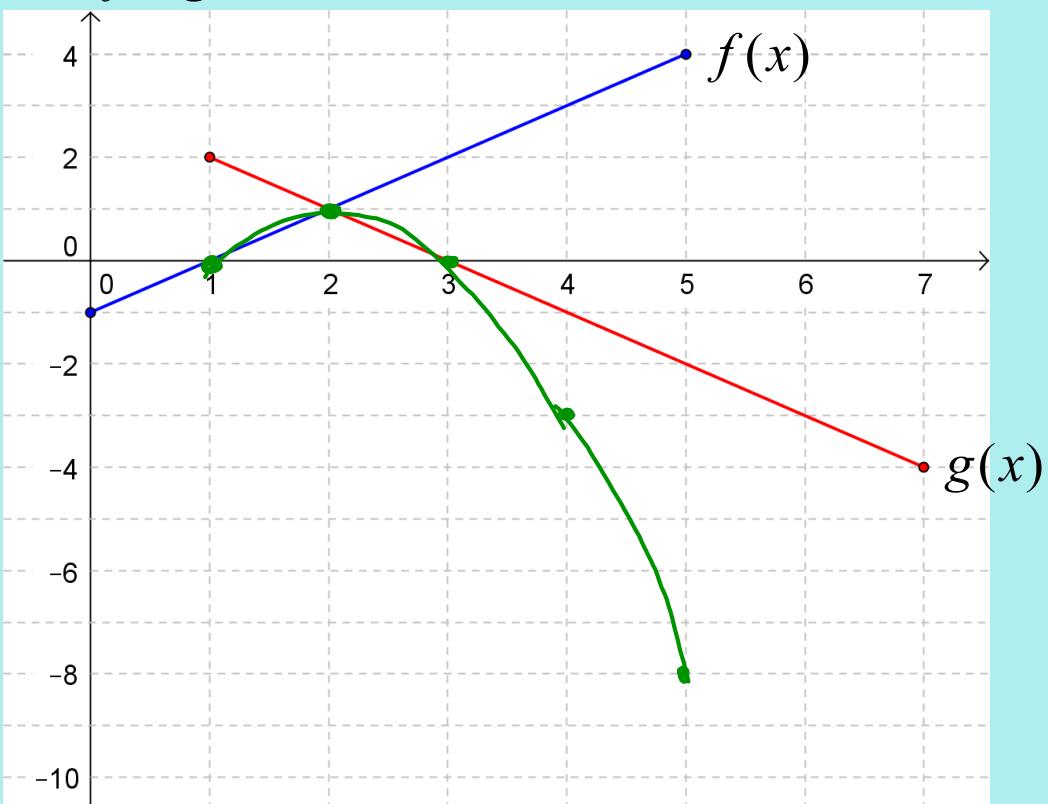
$$D_g = \{2, 3, 4\}$$

$$D_{f \times g} = \{2, 3\}$$

$$f \times g = \{(2, 10), (3, 21)\}$$

Jan 6-8:57 AM

Graph  $f \times g$



Jan 6-2:10 PM

$$\text{Ex.2 Given } f(x) = \log(2-x) \\ g(x) = \sqrt{x+3}$$

determine  $(f \times g)(x)$  and state the domain.

$$\text{for } D_f: 2-x > 0 \quad \text{for } D_g: x+3 \geq 0 \\ -x > -2 \quad \quad \quad x \geq -3 \\ x < 2 \quad \quad \quad$$

$$D_f \leftarrow \begin{array}{c} \text{---} \\ | \\ \text{---} \end{array} \quad D_g \rightarrow \begin{array}{c} \text{---} \\ | \\ \text{---} \end{array}$$

-3                            2

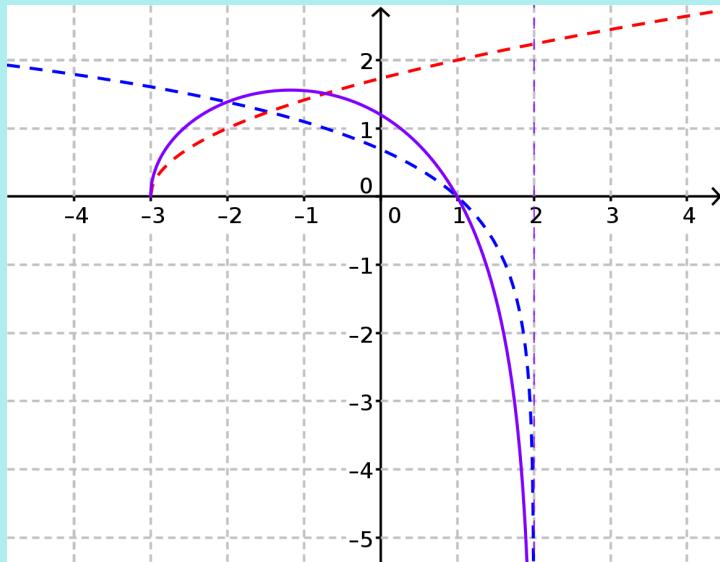
$$D_{f \times g} = \{ x \in \mathbb{R} \mid -3 \leq x < 2 \}$$

$$f \times g = (\log(2-x))(\sqrt{x+3}) \\ = \sqrt{x+3} \log(2-x)$$

Dec 18-3:17 PM

Ex.2 Given  $f(x) = \log(2 - x)$   
 $g(x) = \sqrt{x + 3}$

determine  $(f \times g)(x)$  and state the domain.



Dec 18-3:17 PM

Assigned Work:

p.537 # 1ae, 3, 4bef, 5bef, 6ef, 8ad, 11, 12

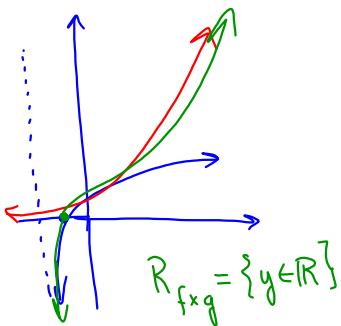
$$S(f) \quad f(x) = \log(2 - x)$$

$$g(x) = 2^x$$

$$(f \times g)(x) = 2^x \log(2 - x)$$

$$D_f: x+4>0 \quad x>-4 \quad D_g: \{x \in \mathbb{R}\}$$

$$\{x \in \mathbb{R} \mid x > -4\} \quad D_{f \times g} = \{x \in \mathbb{R} \mid x > -4\}$$



Jan 6-9:35 AM

8(a)  $f \times g = \frac{\sec x}{(x-7)(x+2)}$

$\rightarrow \frac{1}{\cos x}$

$x \neq 7, -2$

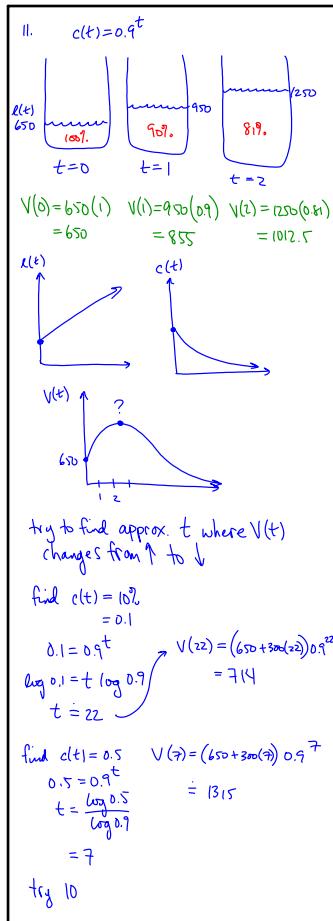
$\text{Graph of } \frac{1}{\cos x}$  (red curve) with vertical asymptotes at  $x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}$ . The graph has discontinuities at  $x = \frac{\pi}{2} + n\pi, n \in \mathbb{Z}$ .

$D_{f \times g} = \left\{ x \in \mathbb{R} \mid x \neq -2, 7, \frac{\pi}{2} + n\pi, n \in \mathbb{Z} \right\}$

$t_n = a + (n-1)d$

$$\begin{aligned} &= \frac{\pi}{2} + (n-1)\pi \\ &= \frac{\pi}{2} + n\pi - \pi \\ &= -\frac{\pi}{2} + n\pi \end{aligned}$$

Jan 8-10:45 AM



Jan 8-10:51 AM