Composition of Functions

Suppose you were asked to graph $y = 2^{\sin x}$.

To make matters worse, suppose your calculator could only perform one operation at a time (i.e., you could perform the exponential operation, or the sine operation, but not both).

How would you get the points for your graph?

x	① $\sin x$	$2^{\sin x}$
-3	$\sin(-3) \doteq -0.141$	$2^{-0.141} \doteq 0.907$
-2	$\sin(-2) \doteq -0.909$	$2^{-0.909} = 0.532$
-1	$\vdots$	
0		
1		
2		
3		

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Composition of Functions

Jan 9/2019

One way to view a function is as a machine, with an input (the independent variable, x) and an output (the dependent variable, y).

$$x \longrightarrow \boxed{f(x)} \longrightarrow y$$

input machine output

It is possible to connect multiple functions (machines) together, so the output of the first is the input to the second.

$$x_1 \longrightarrow f(x) \longrightarrow y_1 = x_2 \longrightarrow g(x) \longrightarrow y_2$$

1st input 1st output is 2nd input 2nd output

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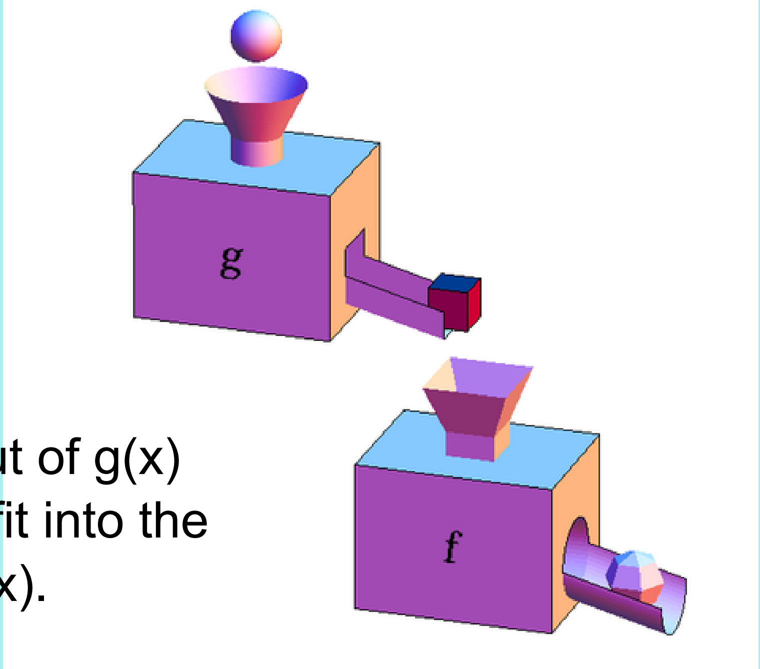
$$f(g(x))$$



The output of g(x) needs to fit into the input of f(x).

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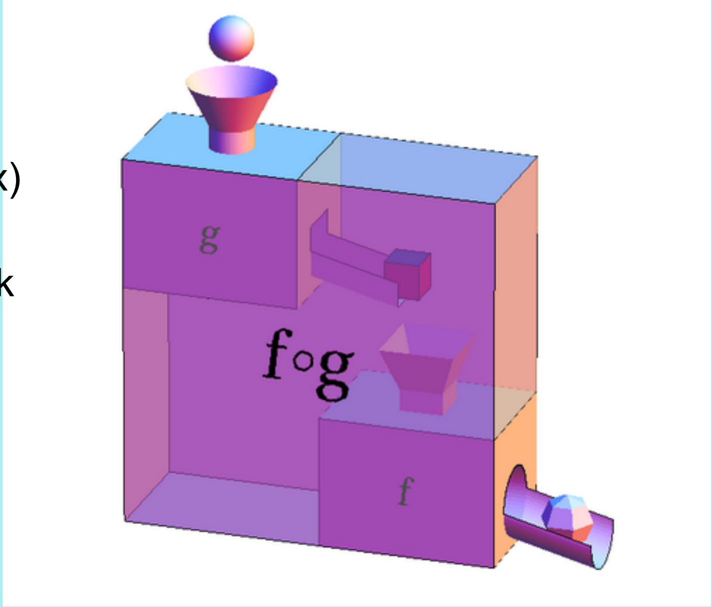
$$f(g(x))$$

The output of g(x) must fit into f(x).

If not, it will break the machine (no valid output).



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A <u>composition</u> of functions occurs when the <u>argument</u> of a function is another function.

$$\underbrace{(f \circ g)(x)}_{\text{f composed with g, "f of g of x"}} = \underbrace{f}_{\text{outer function (calculate 2nd)}}\big(\underbrace{g(x)}_{\text{inner function (calculate 1st)}}\big)$$

Ex.1 Given $f(x)=\sqrt{x}$ and $g(x)=x^2-4$

Express:

(a) $(f \circ g)(x)$

$= f(g(x))$

$= f(x^2-4)$

$= \sqrt{x^2-4}$

(b) $(g \circ f)(x)$

$= g(f(x))$

$= g(\sqrt{x})$

$= (\sqrt{x})^2 - 4$

$= x - 4, \ x \geq 0$

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Ex.1 Given $f(x)=\sqrt{x}$ and $g(x)=x^2-4$

(c) create a table of values for $(f \circ g)(x)$

(d) determine the domain of $f \circ g$

x_1	$g(x)=x^2-4$	→	x_2	$f(x)=\sqrt{x}$
-3	$(-3)^2-4 = 5$		5	$\sqrt{5}$
-2	$(-2)^2-4 = 0$		0	$\sqrt{0} = 0$
-1	$(-1)^2-4 = -3$		-3	$\sqrt{-3}$ inad
0	$0^2-4 = -4$		-4	$\sqrt{-4}$ inad
1	$1^2-4 = -3$		-3	$\sqrt{-3}$ inad
2	$2^2-4 = 0$		0	0
3	$3^2-4 = 5$		5	$\sqrt{5}$

(d) $D_{f \circ g} = \{x \in \mathbb{R} \mid x \leq -2, x \geq 2\}$

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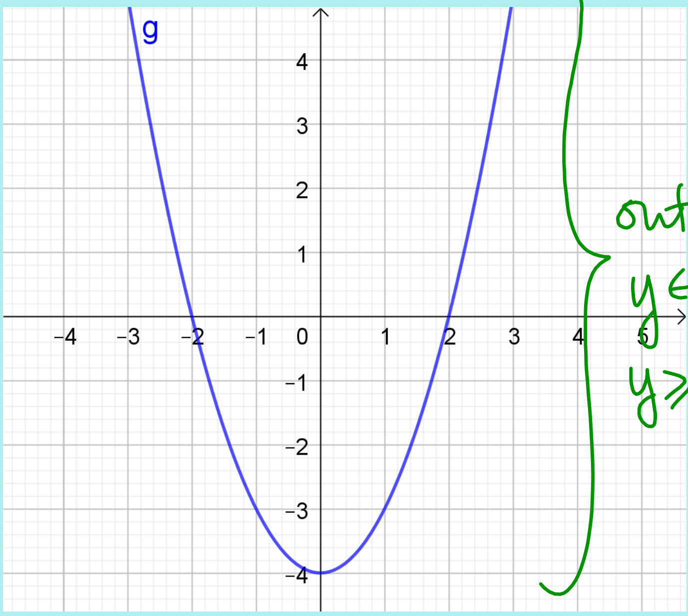

output

$y \in \mathbb{R}$

$y \geq -4$

input

$x \in \mathbb{R}$

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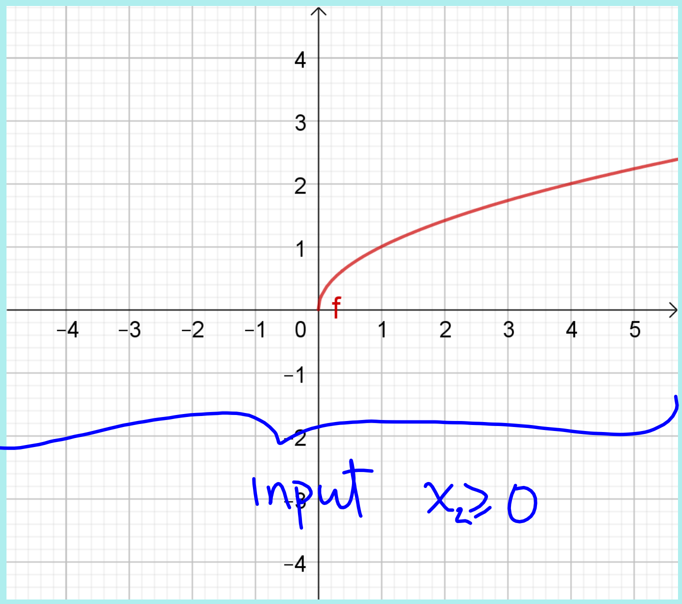
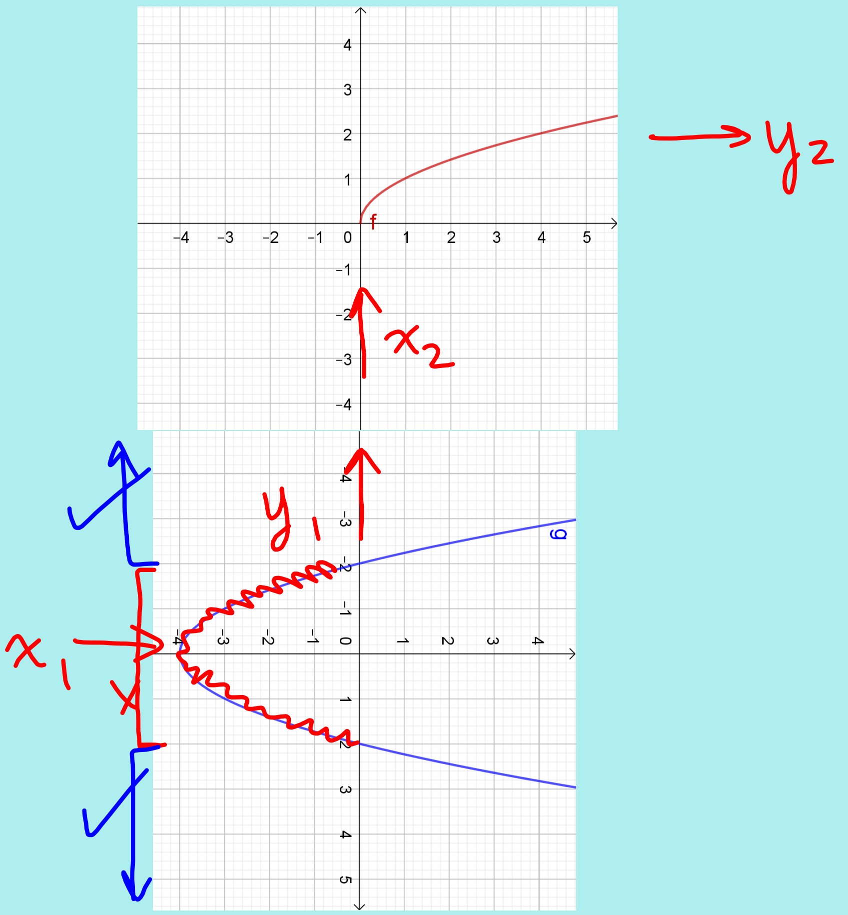


input $x_2 \geq 0$

input from $f(x)$

$y_1 = x_2 \geq -4$

Jan 9-9:03 AM



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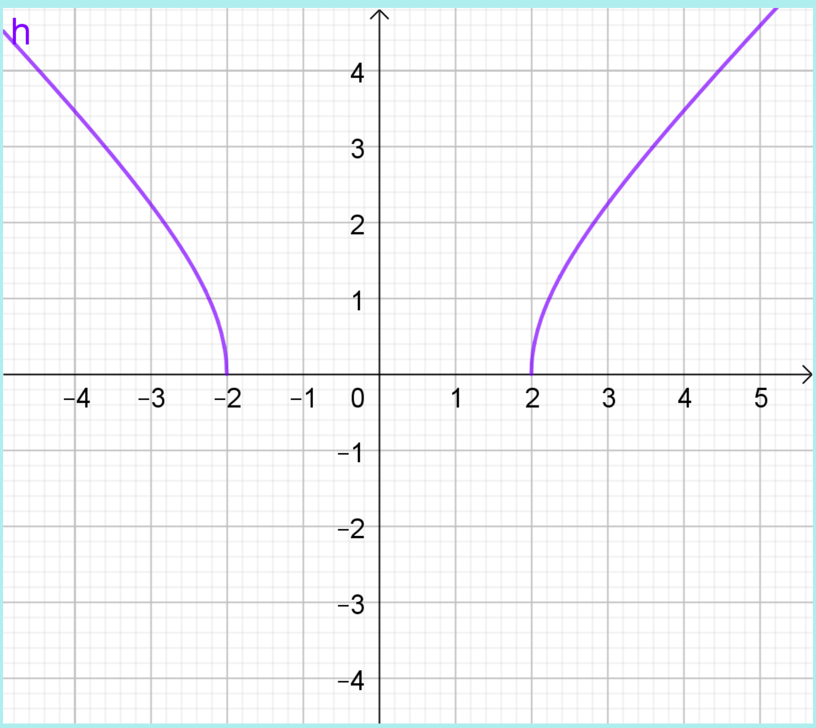
$$(f \circ g)(x) = \sqrt{x^2 - 4}$$



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Ex.1 Given $f(x)=\sqrt{x}$ and $g(x)=x^2-4$

(e) determine the domain of $f \circ g$ algebraically.

When determining the domain of $f \circ g$ algebraically:
(1) determine the domain (restrictions) of the outer function.
(2) create an equation or inequality using the inner function.
(3) solve for the restrictions on the inner function.

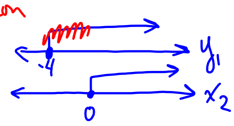
inner outer

$x_1 \rightarrow g(x) \rightarrow y_1 = x_2 \rightarrow f(x) \rightarrow y_2$

$x_1 \in \mathbb{R}$ $\quad$ $\boxed{y_1 \geq -4 \cap x_2 \geq 0}$ $\quad$ $y_2 \geq 0$

intersection

$x_2, y_1 \geq 0$



$x^2-4 \geq 0$

$(x-2)(x+2) \geq 0$

	-2		2	
x+2	−	+	+	
x−2	−	−	+	
	+ ✓	− ✗	+ ✓	

$(-\infty, -2]$ $\quad$ $[2, \infty)$

$D_{f\circ g} = \{x \in \mathbb{R} \mid x \leq -2, x \geq 2\}$

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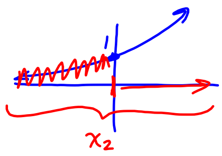
Assigned Work:

p.552 # 1, 2abf, 3, 5aef, 6def, 7cf, 10, 13

6(d) $f(x) = 2^x$, $g(x) = \sqrt{x-1}$

$f \circ g = f(g(x))$
$= f(\sqrt{x-1})$
$= 2^{\sqrt{x-1}}$

$g \circ f = \sqrt{2^x - 1}$



$f \circ g$:

inner outer

$x_1 \rightarrow g(x) \rightarrow y_1 = x_2 \rightarrow f(x) \rightarrow y_2$

$x-1 \geq 0$ $\quad$ $\sqrt{x-1}$ $\quad$ $y_1 \geq 0 \cap x_2 \in \mathbb{R}$ $\quad$ 2^x $\quad$ $y_2 > 0$

$x \geq 1$

$x \geq 1$ $\quad$ $y_1 \geq 0$, $x_2 \geq 0$ $\quad$ $y_2 \geq 1$

$D_{f\circ g} = \{x \in \mathbb{R} \mid x \geq 1\}$

$R_{f\circ g} = \{y \in \mathbb{R} \mid y \geq 1\}$

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6(a) $f(x)=10^x$ $\quad g(x)=\log x$

$f\circ g = 10^{\log x}$ $\quad g\circ f = \log 10^x$

$f\circ g$:

$x_1 \to \log x \to y_1 = x_2 \to 10^x \to y_2$

$x_1>0$ $\quad y_1\in\mathbb{R} \cap x_2\in\mathbb{R}$ $\quad y_2>0$

$x_1>0$ $\quad y_1\in\mathbb{R}$ $\quad x_2\in\mathbb{R}$ $\quad y_2>0$

$D_{f\circ g}=\{x\in\mathbb{R}\mid x>0\}$ $\quad R_{f\circ g}=\{y\in\mathbb{R}\mid y>0\}$

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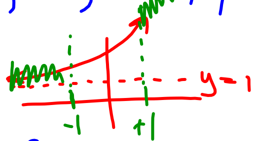
6(f) $f(x)=\sin x$ $\quad g(x)=5^{2x}+1$

$f\circ g=\sin(5^{2x}+1)$ $\quad g\circ f=5^{2\sin x}+1$

$g\circ f$:



$x_1 \to \sin x \to y_1 = x_2 \to 5^{2x}+1 \to y_2$

$x_1\in\mathbb{R}$ $\quad -1\le y_1\le 1 \cap x_2\in\mathbb{R}$ $\quad y_2>1$

$x_1\in\mathbb{R}$ $\quad -1\le y_1\le 1$ $\quad -1\le x_2\le 1$

$\min = g(-1) = 5^{-2}+1 = \frac{26}{25}$

$D_{g\circ f}=\{x\in\mathbb{R}\}$

$R_{g\circ f}=\{y\in\mathbb{R}\mid \frac{26}{25}\le y\le 26\}$

$\max = g(1) = 5^2+1 = 26$

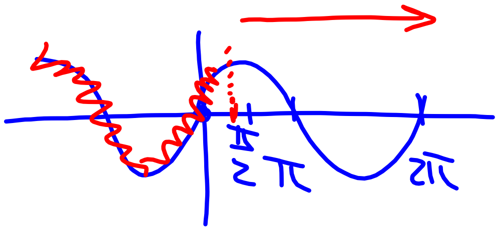
Jan 14-10:45 AM

6(f) $f \circ g = \sin(5^{2x}+1)$

$x_1 \to 5^{2x}+1 \to y_1 = x_2 \to \sin x \to y_2$

$x_1 \in \mathbb{R}$ ✓ $\quad y_1 > 1$ ✓ $\cap\ x_2 \in \mathbb{R} \quad -1 \le y_2 \le 1$

$x_2 > 1$



$-1 \le y_2 \le 1$

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