Feb 8/2019

Unit 1 - Quadratic Functions & Relations

Finding Max/Min Values by Completing the Square

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Finding Max/Min Values by Completing the Square

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Standard Form $y = ax^2 + bx + c$ $\xrightarrow{\text{complete the square}}$ Vertex Form $y = a(x-h)^2 + k$

Vertex is (h, k)

a > 0: opens up (has a minimum)
a < 0: opens down (has a maximum)

k is the optimal value (max or min value)

h is the x-value where the max/min occurs
$x = h$ is the equation of the axis of symmetry

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Completing the Square Using Fractions:

Ex.1 Complete the square:

(a) $y = 3x^2 + 2x - 11$

$y = 3\left[x^2 + \frac{2}{3}x\right] - 11$

$= 3\left[\underbrace{x^2 + \frac{2}{3}x + \frac{1}{9}} - \frac{1}{9}\right] - 11$

$= 3\left[\left(x + \frac{1}{3}\right)^2 - \frac{1}{9}\right] - 11$

$= 3\left(x + \frac{1}{3}\right)^2 - \frac{1}{3} - 11 \times \frac{3}{3}$

$= 3\left(x + \frac{1}{3}\right)^2 - \frac{1}{3} - \frac{33}{3}$

$= 3\left(x + \frac{1}{3}\right)^2 - \frac{34}{3}$

$V\left(-\frac{1}{3}, -\frac{34}{3}\right)$

$\frac{2}{3} \div 2$

$= \frac{\cancel{2}^{1}}{3} \times \frac{1}{\cancel{2}_{1}} = \frac{2}{6}$

$= \frac{1}{3}$

$\left(\frac{1}{3}\right)^2 = \left(\frac{1}{3}\right)\left(\frac{1}{3}\right)$

$= \frac{1}{9}$

$3\left(-\frac{1}{9}\right) = \frac{\cancel{3}^{1}}{1}\left(\frac{-1}{\cancel{9}_{3}}\right)$

$= \frac{-3}{9}$

$= -\frac{1}{3}$

(b) $y = 5x^2 + 8x - 3$

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(b) $y = 5x^2 + 8x - 3$

$= 5\left[x^2 + \frac{8}{5}x\right] - 3$

$= 5\left[\underbrace{x^2 + \frac{8}{5}x + \frac{16}{25}} - \frac{16}{25}\right] - 3$

$= 5\left[\left(x + \frac{4}{5}\right)^2 - \frac{16}{25}\right] - 3$

$= 5\left(x + \frac{4}{5}\right)^2 - \frac{16}{5} - \frac{15}{5}$

$= 5\left(x + \frac{4}{5}\right)^2 - \frac{31}{5}$

$\frac{8}{5} \div 2 = \frac{8}{5} \times \frac{1}{2}$

$= \frac{4}{5}$

$\left(\frac{4}{5}\right)^2 = \frac{16}{25}$

$\cancel{5}^{1}\left(\frac{-16}{\cancel{25}_{5}}\right) = \frac{-16}{5}$

$-3 \times \frac{5}{5} = \frac{-15}{5}$

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Ex.2 Find the optimal value of $y = 5x - 3x^2 + 4$

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It is permissible, and sometimes preferable, to use terminating decimals (i.e., exact values):

Ex.3 Find the optimal value of $y = -20x^2 + 180x + 4400$

$$-\frac{9}{2} = -4.5$$

$$(-4.5)^2 = 20.25$$

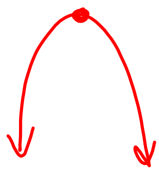
$$y = -20[x^2 - 9x] + 4400$$

$$= -20[\underbrace{x^2 - 9x + 20.25}_{} - 20.25] + 4400$$

$$= -20[(x - 4.5)^2 - 20.25] + 4400$$

$$= -20(x - 4.5)^2 + 405 + 4400$$

$$= -20(x - 4.5)^2 + 4805$$



V(4.5, 4805)

$\therefore$ the optimal value is 4805 (max)

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Homework:

p.115 # 1cegik, 3odd, 7, 14, 16

i

1.(g) $x^2 + 0.8x + \underset{?}{\underline{\underline{c}}}$

$\frac{0.8}{2} = 0.4 \quad (0.4)^2 = 0.16$

$c = 0.16$

or

$x^2 + 0.8x + c$

$= x^2 + \frac{8}{10}x + c$

$= x^2 + \frac{4}{5}x + c$

$\frac{\cancel{4}^{2}}{5} \times \frac{1}{\cancel{2}_{1}} = \frac{2}{5} \quad \left(\frac{2}{5}\right)^2 = \frac{4}{25}$

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3(i) $y = \underline{-2x^2 - 0.8x} - 2$

$= -2\left[x^2 + \underline{0.4x}\right] - 2$

$\frac{0.4}{2} = 0.2 \quad (0.2)^2 = 0.04$

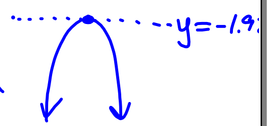
$= -2\left[\underline{x^2 + 0.4x + 0.04} - 0.04\right] - 2$

$= -2\left[(x+0.2)^2 - 0.04\right] - 2$

$= -2(x+0.2)^2 + 0.08 - 2$

$= -2(x+0.2)^2 - 1.92$

$V(-0.2, -1.92)$, opens down



at $x = -0.2$, max of -1.92

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7. let x be the number

$y = 1x^2 - 8x + 35$

$y = \underbrace{x^2 - 8x + 16} - 16 + 35$

$y = (x-4)^2 + 19$

$V(4, 19)$

at $x = 4$, min value of 19

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14. $h = -\frac{1}{2}gt^2 + v_0 t + h_0$

$h = -\frac{1}{2}(9.8)t^2 + (34.3)t + (2.1)$

$h = -4.9t^2 + 34.3t + 2.1$

$h = -4.9[t^2 - 7t] + 2.1$

$h = -4.9[t^2 - 7t + 12.25 - 12.25] + 2.1$

$-\frac{7}{2} = -3.5 \quad (-3.5)^2$

$h = -4.9[(t-3.5)^2 - 12.25] + 2.1$

$h = -4.9(t-3.5)^2 + 60.025 + 2.1$

$h = -4.9(t-3.5)^2 + 62.125$

$V(3.5, 62.125)$

$t \rightarrow$ (b) $\quad h \rightarrow$ (a)

(a) ∴ ______

(b) ∴ ______

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16. $h = -0.004d^2 + 0.14d + 2$

$h = -0.004[d^2 - 35d] + 2$

$= -0.004[d^2 - 35d + 306.25 - 306.25] + 2$

$= -0.004[(d - 17.5)^2 - 306.25] + 2$

$= -0.004(d - 17.5)^2 + 1.225 + 2$

$= -0.004(d - 17.5)^2 + 3.225$

$V(\underset{d}{17.5}, \underset{h}{3.225})$

a/b

$\therefore$ max height is 3.225 m, which occurs 17.5 m away from batter

(c) set $d = 0$, $h = 2$

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