Recall: The simplest quadratic relation is $y = x^2$

On rearranging, it is possible to get answers in the form $x = \pm\sqrt{y}$

With actual values, we might see results such as

$$\sqrt{5} \qquad 3\sqrt{2} \qquad \frac{\sqrt{3}}{2}$$

It is often required to keep answers in this exact form.

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Working With Radicals

index understood to be 2

index of 3

$$\sqrt{5} \qquad\qquad \sqrt[3]{5}$$

radicand
(under radical sign)

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A) Multiplying & Dividing Radicals (same index)

In general, $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$ $\quad \sqrt[3]{xy} = \sqrt[3]{x}\,\sqrt[3]{y}$

and $\dfrac{\sqrt{a}}{\sqrt{b}} = \sqrt{\dfrac{a}{b}}$ *where* $b \neq 0$ $\quad \sqrt{\dfrac{x}{y}} = \dfrac{\sqrt{x}}{\sqrt{y}}$

Ex. Simplify.

(a) $\sqrt{(4)(9)}$

$= \sqrt{36}$
$= 6$

$= \sqrt{4}\sqrt{9}$
$= (2)(3)$
$= 6$

(b) $\sqrt{\dfrac{16}{9}} = \dfrac{\sqrt{16}}{\sqrt{9}}$

$= \dfrac{4}{3}$

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B) Simplifying Radicals

A radical is in its <u>simplest form</u> when:

$\sqrt{8} = \sqrt{4 \times 2}$
$= \sqrt{(2^2)(2)}$

- the radicand has no perfect square factors (other than 1)

$$\sqrt{8} = 2\sqrt{2}$$

- the radicand contains no fractions

$$\sqrt{\frac{5}{4}} = \frac{\sqrt{5}}{\sqrt{4}} = \frac{\sqrt{5}}{2}$$

- no radical appears in the denominator

$$\frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

$\dfrac{2}{\sqrt{3}} \times \dfrac{\sqrt{3}}{\sqrt{3}} = \dfrac{2\sqrt{3}}{\sqrt{9}}$

1 $\quad = \dfrac{2\sqrt{3}}{3}$

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Ex. Simplify

(a) $\sqrt{32} = \sqrt{2 \cdot 16}$
$= \sqrt{2 \cdot 2 \cdot 8}$
$= \sqrt{2 \cdot 2 \cdot 2 \cdot 4}$
$= \sqrt{\underbrace{2 \cdot 2}_{2^2} \cdot \underbrace{2 \cdot 2}_{2^2} \cdot 2}$
$= 4\sqrt{2}$

(b) $2\sqrt{75} = 2\sqrt{25 \cdot 3}$
$= 2\sqrt{25}\sqrt{3}$
$= 2(5)\sqrt{3}$
$= 10\sqrt{3}$

(c) $-3\sqrt{8}$
$= -3(2\sqrt{2})$
$= -6\sqrt{2}$

(d) $\frac{1}{2}\sqrt{\frac{72}{25}} = \frac{1}{2} \cdot \frac{\sqrt{72}}{\sqrt{25}}$
$= \frac{\sqrt{36 \cdot 2}}{2(5)}$
$= \frac{\cancel{6}^{3}\sqrt{2}}{\cancel{10}\,5}$
$= \frac{3\sqrt{2}}{5}$

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C) Adding & Subtracting Radicals

- they must have the same <u>radicand</u>.
- simplify radicals to ensure like terms (same radicand) are revealed.

Ex. Simplify

(a) $4\sqrt{3} - 2\sqrt{5} + 6\sqrt{3} + 5\sqrt{5}$
$= 10\sqrt{3} + 3\sqrt{5}$

let $x = \sqrt{3}$, $y = \sqrt{5}$
$4x - 2y + 6x + 5y$
$= 10x + 3y$
$= 10\sqrt{3} + 3\sqrt{5}$

(b) $2\sqrt{12} - 5\sqrt{27} + 3\sqrt{48} = 2\sqrt{4 \cdot 3} - 5\sqrt{9 \cdot 3} + 3\sqrt{16 \cdot 3}$
$= 4\sqrt{3} - 15\sqrt{3} + 12\sqrt{3}$
$= \sqrt{3}$

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D) Binomial Multiplication of Radicals

Recall: $(a+b)(c+d)=ac+ad+bc+bd$

$(3x+2)(2x-3)$

Ex. Expand & Simplify

$\left(3\sqrt{5}+2\right)\left(2\sqrt{5}-3\right)$ FOIL

$= 6\sqrt{25} - 9\sqrt{5} + 4\sqrt{5} - 6$

$= 30 - 5\sqrt{5} - 6$

$= 24 - 5\sqrt{5}$ ✓

$= -5\sqrt{5} + 24$ ✓ *

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E) Rationalizing the Denominator

A radical is not permitted in the denominator. If the denominator is a binomial, multiply by the conjugate of the denominator.

Given $a\sqrt{b}+c\sqrt{d}$, the conjugate would be $a\sqrt{b}-c\sqrt{d}$

Given $a\sqrt{b}-c\sqrt{d}$, the conjugate would be $a\sqrt{b}+c\sqrt{d}$

Ex. Find the conjugate of each radical

(a) $\sqrt{5}-\sqrt{2}$

C: $\sqrt{5}+\sqrt{2}$

(b) $3\sqrt{5}+2\sqrt{10}$

C: $3\sqrt{5}-2\sqrt{10}$

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Ex. Rationalize the denominator

$$\frac{(4\sqrt{3}-2\sqrt{2})}{(\sqrt{3}-\sqrt{2})} \times \frac{(\sqrt{3}+\sqrt{2})}{(\sqrt{3}+\sqrt{2})}$$

$D = \sqrt{3} - \sqrt{2}$

$C = \sqrt{3} + \sqrt{2}$

$$= \frac{4\sqrt{9} + 4\sqrt{6} - 2\sqrt{6} - 2\sqrt{4}}{\sqrt{9} \underbrace{+\sqrt{6} - \sqrt{6}}_{0} - \sqrt{4}}$$

$(a-b)(a+b)$

$= a^2 - b^2$

$$= \frac{12 + 2\sqrt{6} - 4}{3 - 2}$$

$$= \frac{8 + 2\sqrt{6}}{1}$$

$$= 8 + 2\sqrt{6}$$

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Homework:

p.106 # (1 - 4)(odd)
p.139 # (1 - 7)(odd)

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