Recall: The simplest quadratic relation is $y = x^2$

On rearranging, it is possible to get answers in the form $x = \pm\sqrt{y}$

With actual values, we might see results such as

$$\sqrt{5} \qquad 3\sqrt{2} \qquad \frac{\sqrt{3}}{2}$$

It is often required to keep answers in this exact form.

$$x^2 = 9$$
$$x = \pm\sqrt{9}$$
$$x = \pm 3$$

Feb 6-3:52 PM

## Working With Radicals

Feb 12/2019

index understood to be 2

index of 3

$$\sqrt{5} \qquad \sqrt[3]{5}$$

radicand
(under radical sign)

$$\boxed{\sqrt[x]{y}} \qquad \sqrt[3]{x} = (x)^{\frac{1}{3}}$$

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A) Multiplying & Dividing Radicals

In general, $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$

and $\dfrac{\sqrt{a}}{\sqrt{b}} = \sqrt{\dfrac{a}{b}}$ *where* $b \neq 0$

Ex. Simplify.

(a) $\sqrt{(4)(9)}$

$= \sqrt{36}$
$= 6$

$= \sqrt{4}\sqrt{9}$
$= (2)(3)$
$= 6$

(b) $\sqrt{\dfrac{16}{9}} = \dfrac{\sqrt{16}}{\sqrt{9}}$
$= \dfrac{4}{3}$

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B) Simplifying Radicals

A radical is in its <u>simplest form</u> when:

- the radicand has no perfect square factors (other than 1) $\sqrt{8} = 2\sqrt{2}$

- the radicand contains no fractions $\sqrt{\dfrac{5}{4}} = \dfrac{\sqrt{5}}{\sqrt{4}} = \dfrac{\sqrt{5}}{2}$

- no radical appears in the denominator $\dfrac{2}{\sqrt{3}} = \dfrac{2\sqrt{3}}{3}$

$\sqrt{8} = \sqrt{4 \cdot 2}$
$= \sqrt{4}\sqrt{2}$
$= 2\sqrt{2}$

$\dfrac{2}{\sqrt{3}} \dfrac{\times\sqrt{3}}{\times\sqrt{3}} = \dfrac{2\sqrt{3}}{\sqrt{3 \cdot 3}}$
$= \dfrac{2\sqrt{3}}{3}$
$= \dfrac{2\sqrt{3}}{\sqrt{9}}$
$= \dfrac{2\sqrt{3}}{3}$

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Ex. Simplify

(a) $\sqrt{32}$ $= \sqrt{16 \cdot 2} = 4\sqrt{2}$

$= \sqrt{2 \cdot 16}$

$= \sqrt{2 \cdot 2 \cdot 8}$

$= \sqrt{2 \cdot 2 \cdot 2 \cdot 4}$

$= \sqrt{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2}$

$= 2 \cdot 2\sqrt{2}$

$= 4\sqrt{2}$

$\sqrt{32}$

$= \sqrt{4 \cdot 8}$

$= 2\sqrt{8}$

(b) $2\sqrt{75}$ $= 2\sqrt{25 \cdot 3}$

$2\sqrt{5 \cdot 15}$

$= 2\sqrt{5 \cdot 3 \cdot 5}$

$= 2(5)\sqrt{3}$

$= 10\sqrt{3}$

(c) $-3\sqrt{8}$

$= -3\sqrt{4 \cdot 2}$

$= -3\sqrt{4}\sqrt{2}$

$= -3(2)\sqrt{2}$

$= -6\sqrt{2}$

(d) $\frac{1}{2}\sqrt{\frac{72}{25}}$ $= \frac{1}{2}\left(\frac{\sqrt{72}}{\sqrt{25}}\right)$

$= \frac{1}{2}\left(\frac{\sqrt{36 \cdot 2}}{5}\right)$

$= \frac{1}{\cancel{2}_1}\left(\frac{\cancel{6}^3\sqrt{2}}{5}\right)$

$= \frac{3\sqrt{2}}{5}$

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C) Adding & Subtracting Radicals

- they must have the same radicand.
- simplify radicals to ensure like terms (same radicand) are revealed.

Ex. Simplify

(a) $4\sqrt{3} - 2\sqrt{5} + 6\sqrt{3} + 5\sqrt{5}$

$= 10\sqrt{3} + 3\sqrt{5}$

$4x - 2y + 6x + 5y$

$= 10x + 3y$

(b) $2\sqrt{12} - 5\sqrt{27} + 3\sqrt{48}$

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D) Binomial Multiplication of Radicals

Recall: $(a+b)(c+d) = ac + ad + bc + bd$
F O I L

Ex. Expand & Simplify

$\left(3\sqrt{5}+2\right)\left(2\sqrt{5}-3\right)$

$= (3\sqrt{5})(2\sqrt{5}) + (3\sqrt{5})(-3) + (2)(2\sqrt{5}) + (2)(-3)$

$= 6\sqrt{25} - 9\sqrt{5} + 4\sqrt{5} - 6$

$= 6(5) - 5\sqrt{5} - 6$

$= 24 - 5\sqrt{5}$

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E) Rationalizing the Denominator

A radical is not permitted in the denominator. If the denominator is a binomial, multiply by the <u>conjugate</u> of the denominator.

Given $a\sqrt{b}+c\sqrt{d}$, the conjugate would be $a\sqrt{b}-c\sqrt{d}$

Given $a\sqrt{b}-c\sqrt{d}$, the conjugate would be $a\sqrt{b}+c\sqrt{d}$

Ex. Find the conjugate of each radical

(a) $\sqrt{5}-\sqrt{2}$ (b) $3\sqrt{5}+2\sqrt{10}$

conj: $\sqrt{5}+\sqrt{2}$ conj: $3\sqrt{5}-2\sqrt{10}$

$(x-y)(x+y) = x^2 - y^2$

$(x+y)(x-y) = x^2 - y^2$

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Ex. Rationalize the denominator

$$\frac{4\sqrt{3}-2\sqrt{2}}{\sqrt{3}-\sqrt{2}} \times \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}$$

$$= \frac{(4\sqrt{3})(\sqrt{3}) + (4\sqrt{3})(\sqrt{2}) - (2\sqrt{2})(\sqrt{3}) - (2\sqrt{2})(\sqrt{2})}{(\sqrt{3})^2 - (\sqrt{2})^2}$$

$$= \frac{4\sqrt{9} + 4\sqrt{6} - 2\sqrt{6} - 2\sqrt{4}}{3-2}$$

$$= 12 + 2\sqrt{6} - 4$$

$$= 8 + 2\sqrt{6}$$

$$(\sqrt{3})^2$$

$$= (\sqrt{3})(\sqrt{3})$$

$$= \sqrt{9}$$

$$= 3$$

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Homework:

6i 7a

p.106 # (1 - 4)(odd)
p.139 # (1 - 7)(odd) + 6, 7

p.106 2g 3e 4a
k c
e

p.139 2c 3e 4a 5m
1e

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p.139 6i, 7a

6(i) $\dfrac{4\sqrt{7}}{2\sqrt{14}}$

① brute force

$\dfrac{4\sqrt{7}}{2\sqrt{14}} \times \dfrac{\sqrt{14}}{\sqrt{14}}$

$= \dfrac{4\sqrt{7 \cdot 14}}{2(14)}$

$= \dfrac{\overset{2}{\cancel{4}}\sqrt{98}}{\cancel{28}\,14}$

$= \dfrac{\overset{1}{\cancel{2}}\sqrt{49 \cdot 2}}{\cancel{14}\,7}$

$= \dfrac{\overset{1}{\cancel{7}}\sqrt{2}}{\cancel{7}\,1}$

$= \sqrt{2}$

② $\dfrac{4\sqrt{7}}{2\sqrt{14}}$

$= \dfrac{\overset{2}{\cancel{4}}\overset{1}{\cancel{\sqrt{7}}}}{{}_{1}\cancel{2}\underset{1}{\cancel{\sqrt{7}}}\sqrt{2}}$

$= \dfrac{2}{\sqrt{2}} \times \dfrac{\sqrt{2}}{\sqrt{2}}$

$= \dfrac{\overset{1}{\cancel{2}}\sqrt{2}}{{}_{1}\cancel{2}}$

$= \sqrt{2}$



7(a) $\dfrac{1}{(\sqrt{2}+2)} \times \dfrac{(\sqrt{2}-2)}{(\sqrt{2}-2)}$

$(a+b)(a-b)$
$= a^2 - b^2$

$= \dfrac{\sqrt{2}-2}{2-4}$

$= \dfrac{\sqrt{2}-2}{-2}$ ✓ok

$= \dfrac{-\sqrt{2}+2}{2}$ ✓

$= \dfrac{2-\sqrt{2}}{2}$ ✓

106 2gk

(g) $\frac{\sqrt{20}}{\sqrt{9}} = \frac{\sqrt{20}}{3}$

$= \frac{\sqrt{4 \cdot 5}}{3}$

$= \frac{\sqrt{4}\sqrt{5}}{3}$

$= \frac{2\sqrt{5}}{3}$

(k) $\frac{4\sqrt{2}}{\sqrt{8}} = \frac{4\sqrt{2}}{\sqrt{4 \cdot 2}}$

$= \frac{4\cancel{\sqrt{2}}^{1}}{2\cancel{\sqrt{2}}_{1}}$

$= \frac{4}{2}$

$= 2$

Feb 14-1:59 PM

106 3e

$4\sqrt{3} \times \sqrt{7}$

$= 4\sqrt{3 \cdot 7}$

$= 4\sqrt{21}$

Feb 14-2:01 PM

106 4ace

(a) $\frac{10+15\sqrt{5}}{5} = \frac{5\overbrace{(2+3\sqrt{5})}^{x}}{5}$

$= \frac{\cancel{5}^{1}x}{\cancel{5}_{1}}$

$= x$

$= 2+3\sqrt{5}$

(c) $\frac{6+\sqrt{8}}{2} = \frac{6+\sqrt{4\cdot 2}}{2}$

$= \frac{6+2\sqrt{2}}{2}$

$= \frac{\cancel{2}^{1}(3+\sqrt{2})}{\cancel{2}_{1}}$

$= 3+\sqrt{2}$

(e) $\frac{-10-\sqrt{50}}{5}$

$= \frac{-10-\sqrt{25\cdot 2}}{5}$

$= \frac{-10-5\sqrt{2}}{5}$

$= \frac{5(-2-\sqrt{2})}{5}$

$= -2-\sqrt{2}$

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p. 139 1(e) $8\sqrt{10} - 2\sqrt{10} - 7\sqrt{10}$

$= -\sqrt{10}$

$\sqrt{10}$

$= \sqrt{5\cdot 2}$

2(c) $2\sqrt{2} + 3\sqrt{10} + 5\sqrt{2} - 4\sqrt{10}$

$= 2x + 3y + 5x - 4y$

$= 7\sqrt{2} - \sqrt{10}$

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3e 4a 5m

3(e) $\sqrt{75}+\sqrt{48}+\sqrt{27}$

$=\sqrt{25\cdot 3}+\sqrt{16\cdot 3}+\sqrt{9\cdot 3}$

$=5\sqrt{3}+4\sqrt{3}+3\sqrt{3}$

$=12\sqrt{3}$

4(a) $8\sqrt{7}+2\sqrt{28}$

$=8\sqrt{7}+2\sqrt{4\cdot 7}$

$=8\sqrt{7}+2(2\sqrt{7})$

$=8\sqrt{7}+4\sqrt{7}$

$=12\sqrt{7}$

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5(m) $(\sqrt{6}-\sqrt{2})(\sqrt{6}+\sqrt{2})$

$=(x-y)(x+y)$

① FOIL
② difference of squares

$=x^2-y^2$

$=(\sqrt{6})^2-(\sqrt{2})^2$

$=6-2$

$=4$

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