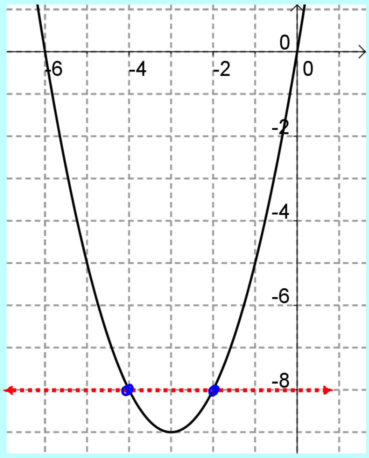
Intersection of Quadratics & Lines

(more solving quadratic equations)

Recall from last class:

Consider $y = x^2 + 6x$, and solve for $y = -8$.



In this example, we were actually solving for the intersection between the parabola and the horizontal straight line.

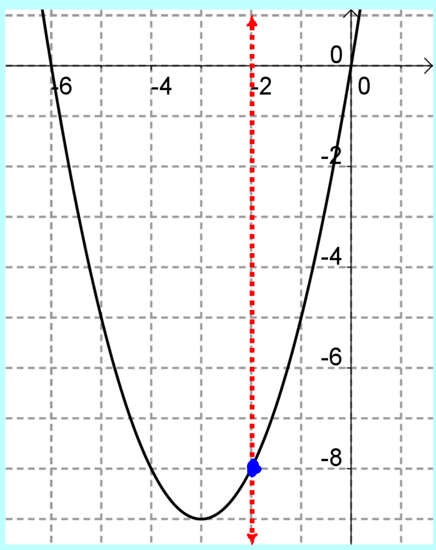
Solutions: (-4, -8) and (-2, -8)

x y x y

Intersection of Quadratics & Lines

(more solving quadratic equations)

Consider $y = x^2 + 6x$, and solve for $x = -2$.



In this example, we solve for the intersection between the parabola and the vertical straight line.

Solution: (-2, -8)

x y

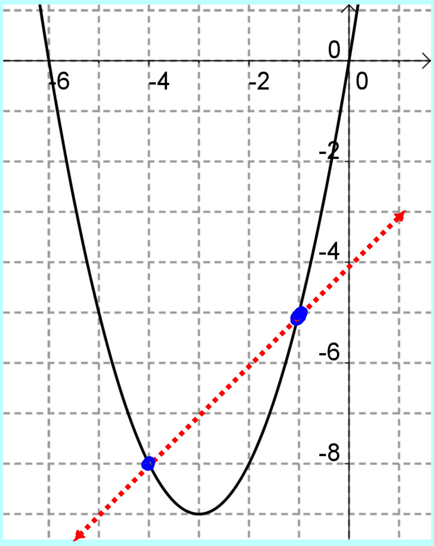
Intersection of Quadratics & Lines

(more solving quadratic equations)

Consider $y = x^2 + 6x$, and solve for $y = x - 4$.

$y = mx + b$

In this example, we solve for the intersection between the parabola and the given straight line.

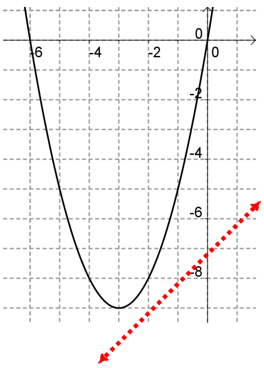


Solutions: (-4, -8) and (-1, -5)

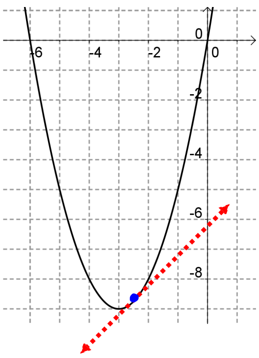
Intersection of Quadratics & Lines

(more solving quadratic equations)

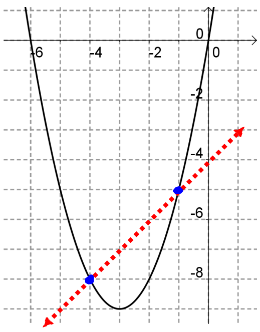
A linear-quadratic system will have zero, one, or two solutions.



No Solution



One Solution (tangent line)



Two Solutions (secant line)

Recall: To solve an equation is to find the value(s) for the variables that satisfy the equation (i.e., make it true)

$P(x, y)$

Given a quadratic relation, $\boxed{y = Ax^2 + Bx + C}$

and a linear relation, $y = mx + k$

the solution will be the point(s) where the parabola and straight line intersect.

$$y = Ax^2 + Bx + C \quad (1) \qquad y = mx + k \quad (2)$$

Solve the system of equations using the fact that $y = y$

$$y = y$$

$$Ax^2 + Bx + C = mx + k$$

$$\vdots$$

$$ax^2 + bx + c = 0$$

factor?
CTS?
GQF?

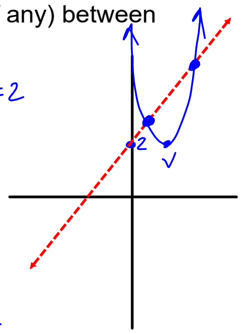
Sub the x-values from the solution(s) into either original relation to find the corresponding y-values.

Ex.1 Find the points of intersection (if any) between

$y = 2(x-1)^2 + 2$ and $y = x + 2$

$V(1, 2)$

$m = 1$, y-int $= 2$



want $y_1 = y_2$

$2(x-1)^2 + 2 = x + 2$

$2(x^2 - 2x + 1) \cancel{+2} = x \cancel{+2}$

$2x^2 - 4x + 2 - x = 0$

$2x^2 - 5x + 2 = 0$

S -5
P 4
I $-1, -4$

$\underbrace{2x^2 - x} \underbrace{- 4x + 2} = 0$

$x(2x-1) - 2(2x-1) = 0$

$(2x-1)(x-2) = 0$

$2x - 1 = 0$
$x = \frac{1}{2}$

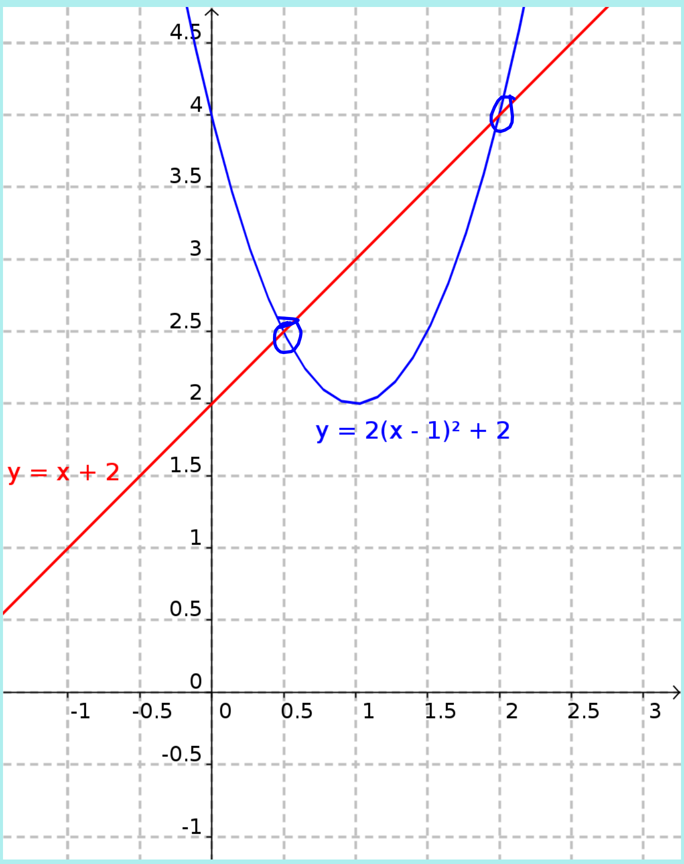
$x = 2$

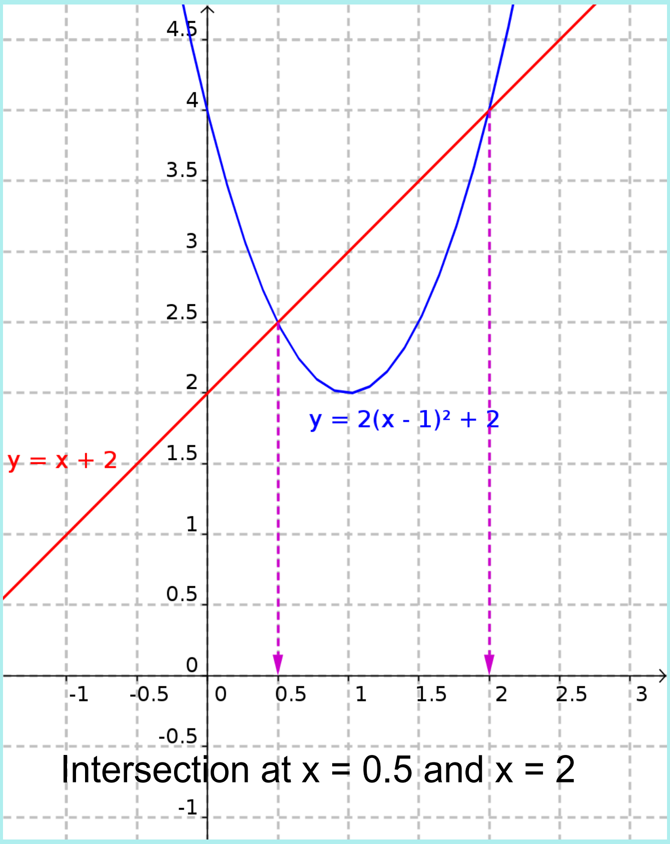
Sub x' into $y = x + 2$

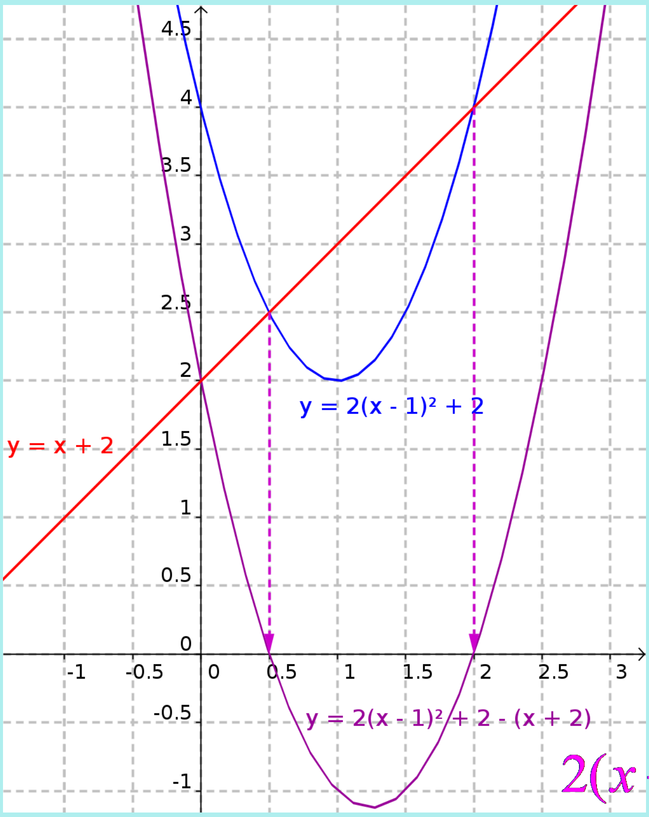
$x = \frac{1}{2}$, $y = \frac{1}{2} + 2$
$= \frac{1}{2} + \frac{4}{2}$
$= \frac{5}{2}$

$x = 2$, $y = 2 + 2$
$= 4$

$\therefore$ points of intersection are
$\left(\frac{1}{2}, \frac{5}{2}\right)$ and $(2, 4)$







Intersection of line (red) and parabola (blue) is equivalent to finding the zeroes of the parabola formed by rearranging the equation (purple)

$$2(x-1)^2+2=x+2$$

$$2(x-1)^2+2-(x+2)=0$$

$$2x^2-5x+2=0$$

Ex.2 Determine the equations of the lines that have a slope of 2 that intersect $y = x(6-x)$
(a) once
(b) twice
(c) never

only one value of k

$y = mx + k$

slope y-int

$y = 2x + k$

$x(6-x) = 2x + k$

$6x - x^2 = 2x + k$

$0 = \underbrace{x^2 - 4x + k}_{ax^2 + bx + c}$

one solution : $D = 0$

$b^2 - 4ac = 0$

$(-4)^2 - 4(1)(k) = 0$

D → $16 - 4k = 0$

$16 = 4k$

$\boxed{k = 4}$

test $k > 4$

$D = 16 - 4k$

$= 16 - 4(5)$

$= 16 - 20$

$= -5$

$D < 0$, no sol

$k < 4$

$D = 16 - 4k$

$= 16 - 4(3)$

$= 16 - 12$

$= 4$

$D > 0$, two sol

(a) $y = 2x + 4$

(b) $y = 2x + k$, $k < 4$

(c) $y = 2x + k$, $k > 4$

Assigned Work:

worksheet