Families of Quadratic Relations

A group of quadratic relations which all share a common characteristic is called a family of quadratics.

The most common characteristics of interest are:

1. Zeroes ✓
2. Vertex ✓
3. y-intercept ✓

same zeroes

$$y = 3(x-2)(x+5)$$

$$y = -(x+5)(x-2)$$

$$y = x^2 + 3x - 10$$

$$= (x+5)(x-2)$$

same y-intercepts

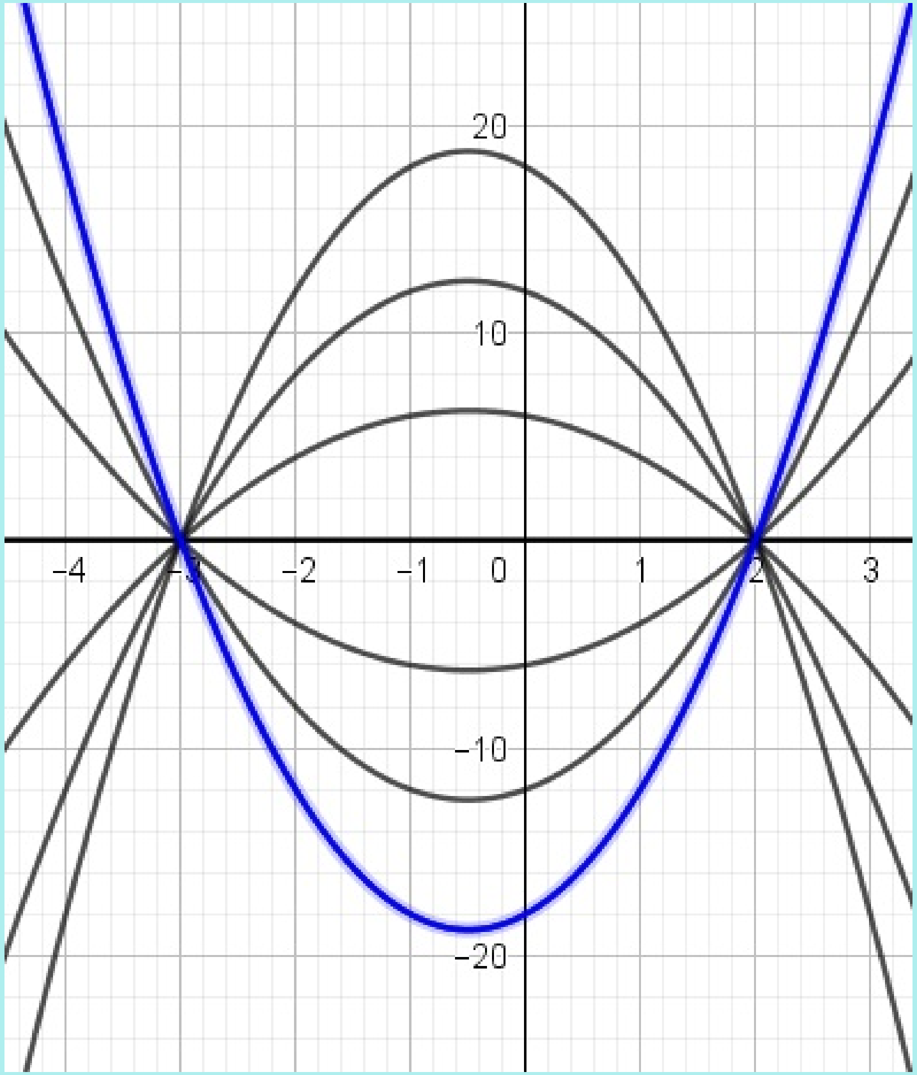
$$y = x^2 + 3x - 10$$

$$y = -5x^2 - x - 10$$

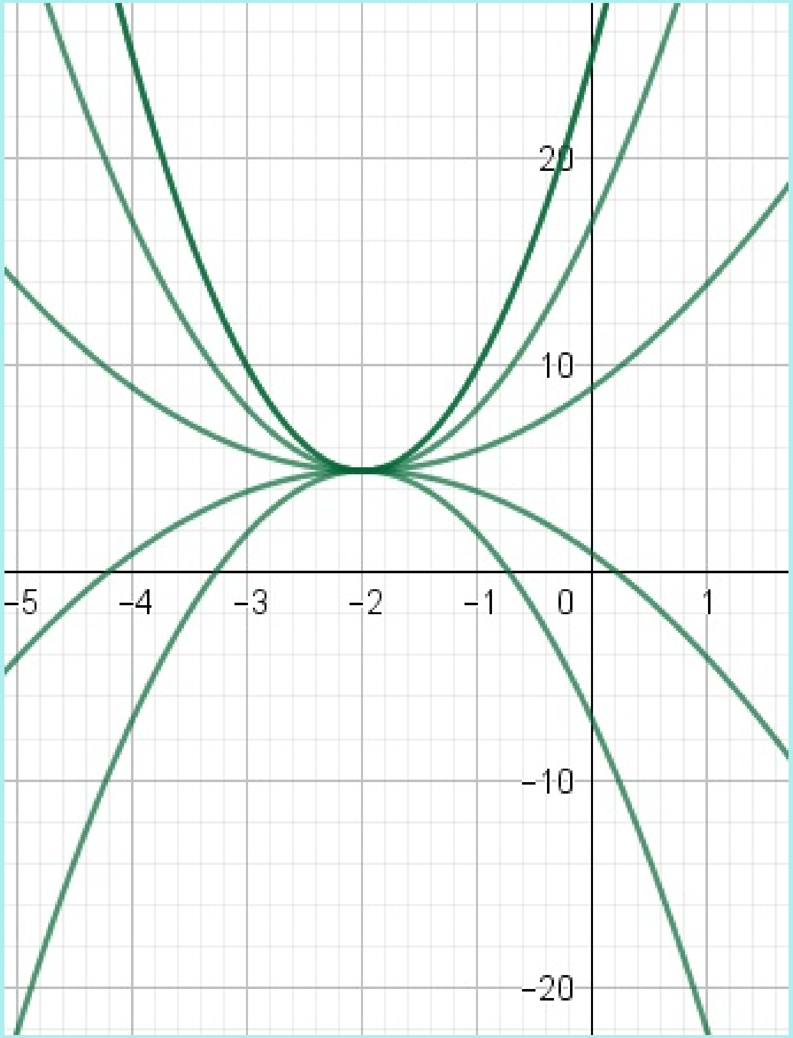
$$y = 2(x+3)^2 - 28$$

Note: The equations may not be in the same form.

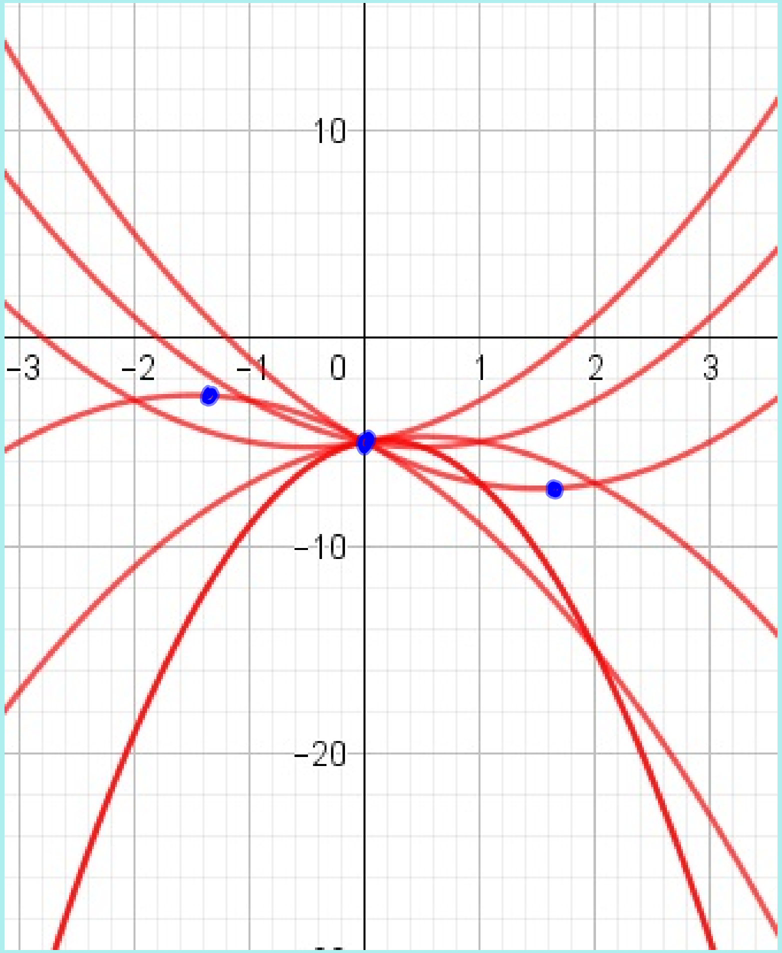
Family of Quadratics - Same Zeroes



Family of Quadratics - Same Vertex



Family of Quadratics - Same y-intercept



Ex. Determine the family of quadratics common to:

$y = -(x-6)(x+2)$ → zeroes

$y = 3(x-2)^2 + 16$ → vertex

$y = -2x^2 + 8x + 8$ → y-intercept

How can we test for the same vertex, zeroes, y-int?

What is the easiest place to start?

How to test for same family of quadratics:

std ⇅ vertex ⇄ factor ⇅ std

1. Change all relations to same form. ✓

2. Identify key points or characteristics, and test them in each relation using substitution.

y-intercepts:

- set x=0
- compare y-int   same ?

zeroes:

$P_1(x_1, 0)$

$P_2(x_2, 0)$

- find zeroes for one relation
- set x=s and x=t to test other relations
- in all tests, should get y=0

vertex:

- find vertex, V(h,k), for one relation
- set x=h in other relations
- in all tests, should get y=k

Ex. Determine the family of quadratics common to:

$y = -(x-6)(x+2)$ A

$y = 3(x-2)^2 + 16$ B

$y = -2x^2 + 8x + 8$ C

C y-int at $(\underset{x}{0}, 8)$

B set $x = 0$

$y = 3(0-2)^2 + 16$
$= 12 + 16$
$= 28$ ✗

A $y = -1(0-6)(0+2)$
$= 12$ ✗

∴ not a family of y-int

A $(6, \underline{0})$ $(-2, \underline{0})$

test B sub $x = 6$ sub $x = -2$

$y = 3(x-2)^2 + 16$
$= 3(6-2)^2 + 16$
$= 48 + 16$
$= 64$ ✗

∴ not common zeroes.

B $V(\underset{x}{2}, \underset{y}{16})$

A : $y = -(x-6)(x+2)$
$= -(2-6)(2+2)$
$= -(-4)(4)$
$= 16$ ✓

C $y = -2x^2 + 8x + 8$
$= -2(2)^2 + 8(2) + 8$
$= -8 + 16 + 8$
$= 16$ ✓

∴ it is a family with common vertex.

Ex. Determine the equation of the quadratic relation, in standard form, that has roots of $1+\sqrt{5}$ and $1-\sqrt{5}$ and passes through:

(a) (2, 5)
(b) (2, 10) } family of roots/zeroes

$$y = a(x-s)(x-t)$$
$$y = a(x-(1+\sqrt{5}))(x-(1-\sqrt{5}))$$
$$y = a(\underbrace{x-1}_{p}-\underbrace{\sqrt{5}}_{q})(\underbrace{x-1}_{p}+\underbrace{\sqrt{5}}_{q})$$
$$(p-q)(p+q)$$
$$y = a(p^2-q^2)$$
$$y = a((x-1)^2-(\sqrt{5})^2)$$
$$y = a(x^2-2x+1-5)$$
$$y = \underset{?}{a}(x^2-2x-4)$$

(a) $P(2,5)$ (x, y)

$$5 = a(2^2-2(2)-4)$$
$$5 = a(4-4-4)$$
$$5 = -4a$$
$$a = -\frac{5}{4}$$

$$y = \frac{-5}{4}(x^2-2x-4)$$
$$y = \frac{-5}{4}x^2 + \frac{5(\cancel{2}^{1}x)}{\cancel{4}_{2}} + \frac{5(\cancel{4}^{1})}{\cancel{4}_{1}}$$
$$\boxed{y = \frac{-5}{4}x^2 + \frac{5x}{2} + 5}$$

(b) $P(2,10)$

$$y = a(x^2-2x-4)$$
$$10 = a(4-4-4)$$
$$10 = -4a$$
$$a = \frac{-10}{4}$$
$$a = \frac{-5}{2}$$

$$y = \frac{-5}{2}(x^2-2x-4)$$
$$\boxed{y = -\frac{5}{2}x^2+5x+10}$$

Assigned Work:

worksheet