

## Transformations of Functions

Recall: In Gr.10, we explored transformations of a quadratic relation. For consistency, we use vertex form.

$$y = \underline{a} (x - h)^2 + k$$

What are the possible transformations?

(read from left to right)

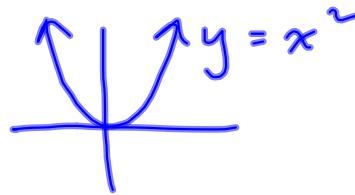
①  $a$ : v. reflection?  
vertical scaling  $\rightarrow$  stretch  
 $\rightarrow$  compression

②  $h$ :  $x_{\text{vertex}}$  horizontal shift left/right

③  $k$ :  $y_{\text{vertex}}$  vertical shift up/down

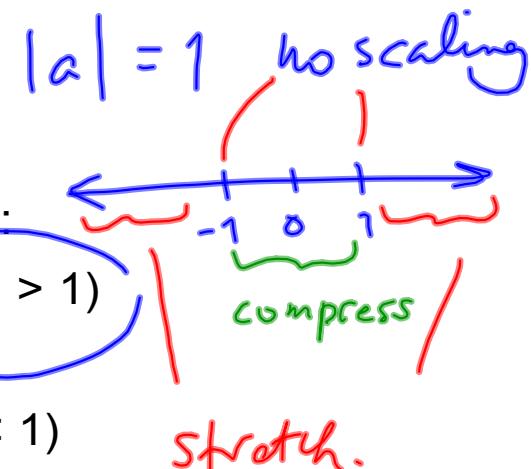
$$y = a(x - h)^2 + k$$

1&2      3      4



Summary of possible transformations (in order):

1. Vertical Reflection:  $a < 0$



2. Vertical Scaling (stretch or compress):

- stretch when  $a < -1$  or  $a > 1$  (or  $|a| > 1$ )
- parabola is thinner
- compress when  $-1 < a < 1$  (or  $|a| < 1$ )
- parabola is wider

3. Horizontal Translation (shift left or right) by  $h$

4. Vertical Translation (shift up or down) by  $k$

Using function notation,

$$y = x^2 \quad \text{becomes} \quad y = f(x)$$

$$y = a(x-h)^2 + k \quad \text{becomes} \quad y = af(x-h) + k$$

The transformations produced by a, h, and k can  
be applied to any function.

Start with:

$$f(x) = x^2 \quad [ \times a ]$$

$$af(x) = ax^2$$

$$af(x-h) = a(x-h)^2$$

$$af(x-h) + k = a(x-h)^2 + k$$

Ex.1 State the transformations, in the correct order,

$$y = f(x)$$

$$\text{has undergone to obtain } y = f(x+1) + 3$$



① ②

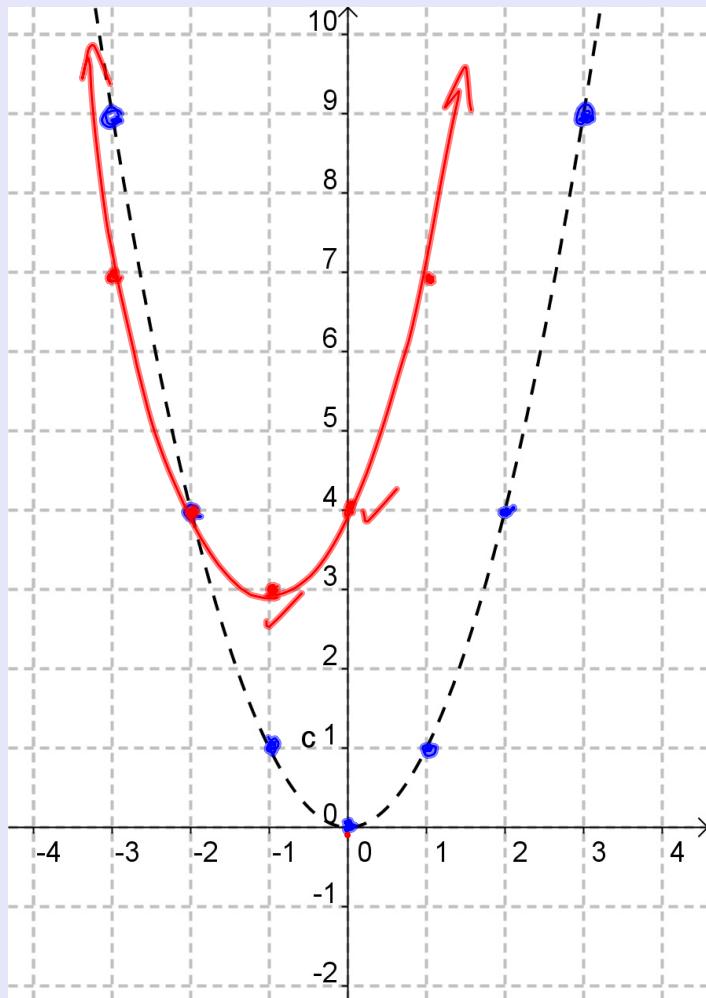
① h. shift left by 1

② v. shift up by 3

Using the previous example, what if the parent function is a quadratic? Then  $f(x) = x^2$ .

Sketch the transformed parabola on the grid below.

(Use a table of values if you want to)



$$y = f(x+1) + 3$$

①    ②

① left 1

② up 3

$$\begin{matrix} (0, 0) \\ x \quad y \end{matrix} \xrightarrow{\textcircled{1}} (-1, 0) \xrightarrow{\textcircled{2}} (-1, 3)$$
$$(1, 1) \longrightarrow (0, 4)$$

What if the parent function is a set of ordered pairs?

$$f(x) = \{(1, 2), (2, -3), (-5, 7)\}$$

what would  $f(x+1)+3$  become?

(Use a table of values and/or graph if you want to!)

① Left by 1 (x)

② up by 3 (y)

$$(1, 2) \xrightarrow{\textcircled{1}} (0, 2) \xrightarrow{\textcircled{2}} (0, 5)$$

$$(2, -3) \rightarrow (1, -3) \rightarrow (1, 0)$$

$$(-5, 7) \rightarrow (-6, 7) \rightarrow (-6, 10)$$

$$f(x+1)+3 = \{(0, 5), (1, 0), (-6, 10)\}$$

What if the parent function is the square root function?

Then  $f(x) = \sqrt{x}$ .

What would  $f(x+1)+3$  become?

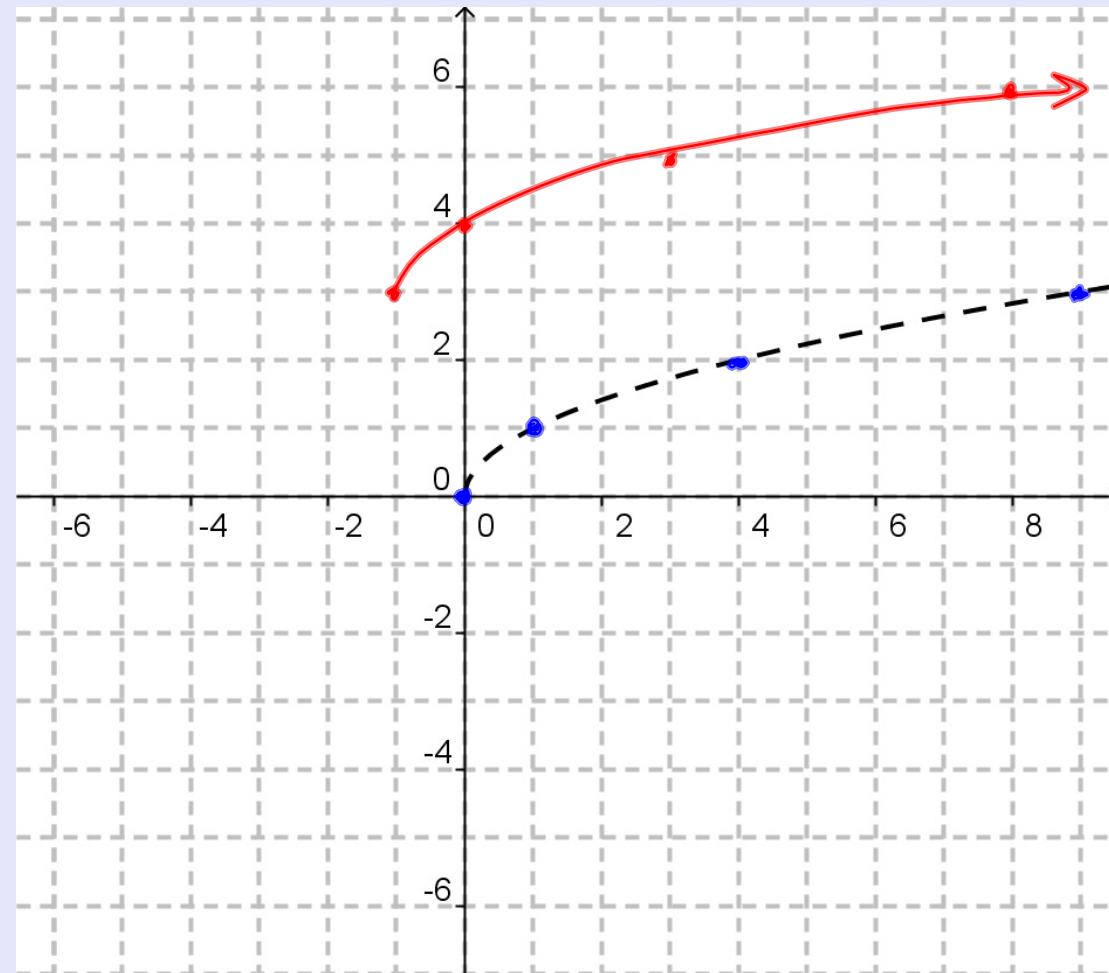
(Use a table of values if you want to)

$$(6, 6) \rightarrow (-1, 3)$$

$$(1, 1) \rightarrow (0, 4)$$

$$(4, 2) \rightarrow (3, 5)$$

$$(9, 3) \rightarrow (8, 6)$$



What if the parent function is the reciprocal function?

Then  $f(x) = \frac{1}{x}$ .

What would  $f(x+1)+3$  become?

(Use a table of values if you want to)

(1, 1)

(-1, -1)

(2, 0.5)

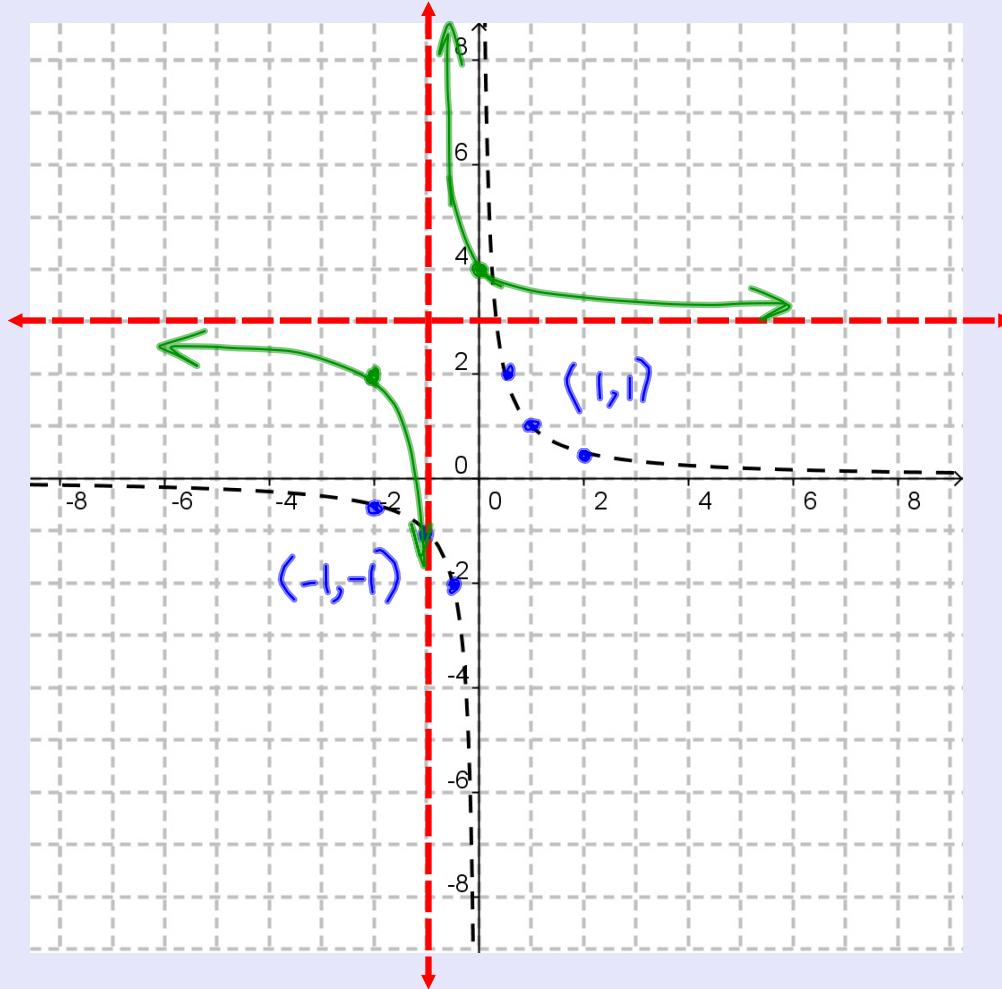
(0.5, 2)

(-2, -0.5)

(-0.5, -2)

VA :  $x = 0 \rightarrow x = -1$

HA :  $y = 0 \rightarrow y = 3$



$$y = 2\sqrt{x} + 3$$

describe the transformations and

apply them to the square root function.

(Use a table of values and/or graph if you want to!)

$$y = 2f(x) + 3$$

①

②

①  $a = 2$

v. stretch by 2

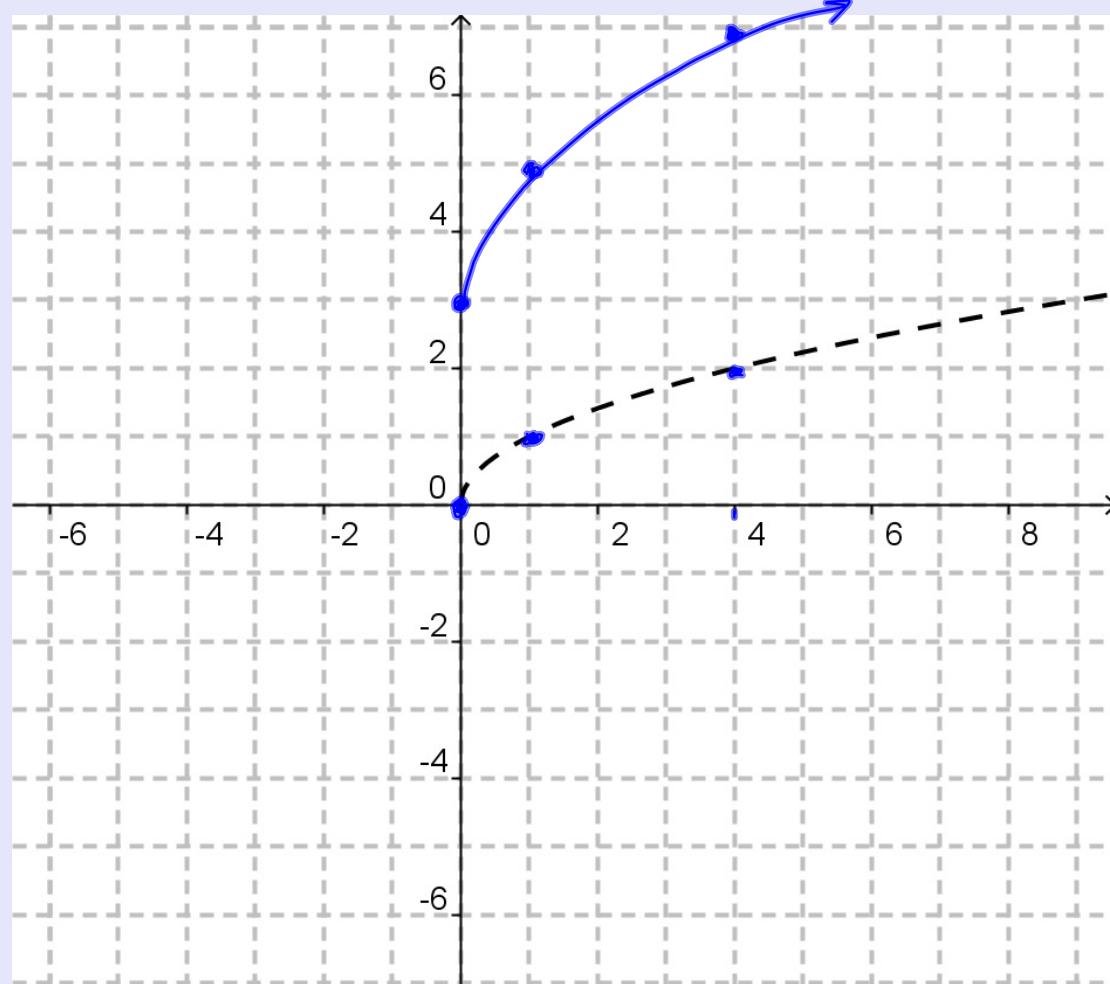
$$y \times 2$$

② v. shift by 3

$$y + 3$$

$$(0,0) \xrightarrow{\textcircled{1}} (0,0) \xrightarrow{\textcircled{2}} (0,3)$$

$$(1,1) \xrightarrow{\textcircled{1}} (1,2) \xrightarrow{\textcircled{2}} (1,5)$$



$$y = 0.4f(x-1)$$

, describe the transformations and

apply them to the reciprocal function.

(Use a table of values and/or graph if you want to!)

$$y = 0.4f(x-1)$$

①      ②

① : v. scaling by 0.4

$$y \times \frac{2}{5}$$

or

$$\frac{4}{10} = \frac{2}{5}$$

v. compress by  $\frac{5}{2} (>1)$

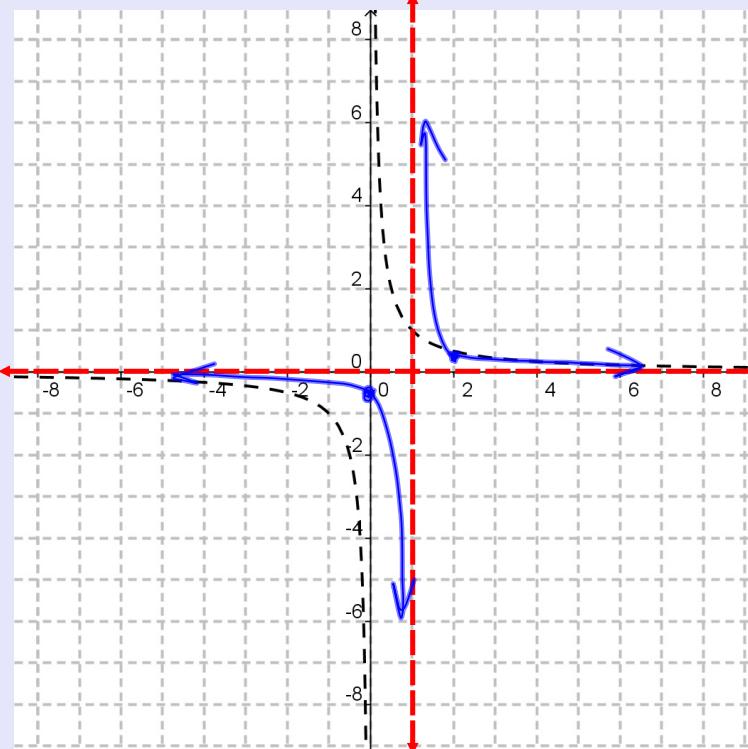
$$y \div \frac{5}{2}$$

??

v. compress by  $\frac{2}{5}$

$$y \times \frac{2}{5}$$

② right by 1



$$\text{VA} : x = 0 \xrightarrow{\textcircled{1}} x = 0 \xrightarrow{\textcircled{2}} x = 1$$

$$\text{HA} : y = 0 \rightarrow y = 0 \rightarrow y = 0.4$$

$$(1, 1) \rightarrow (1, \frac{2}{5}) \rightarrow (2, \frac{2}{5})$$

$$(2, 0.4)$$

$$(-1, -1) \rightarrow (-1, -\frac{2}{5}) \rightarrow (0, -0.4)$$

Ex: The graph on the right shows the function  $y = f(x)$ .  
 $y = -f(x+1) - 2$ , describe the transformations and apply them to the function

(Use a table of values with the key points, if you want to!)

$$y = -f(x+1) - 2$$

①    ②    ③

① : v. reflect

② : shift left by 1

③ : down by 2

$$(-4, -1) \quad (-2, 0)$$

$\downarrow$  ①     $\downarrow$

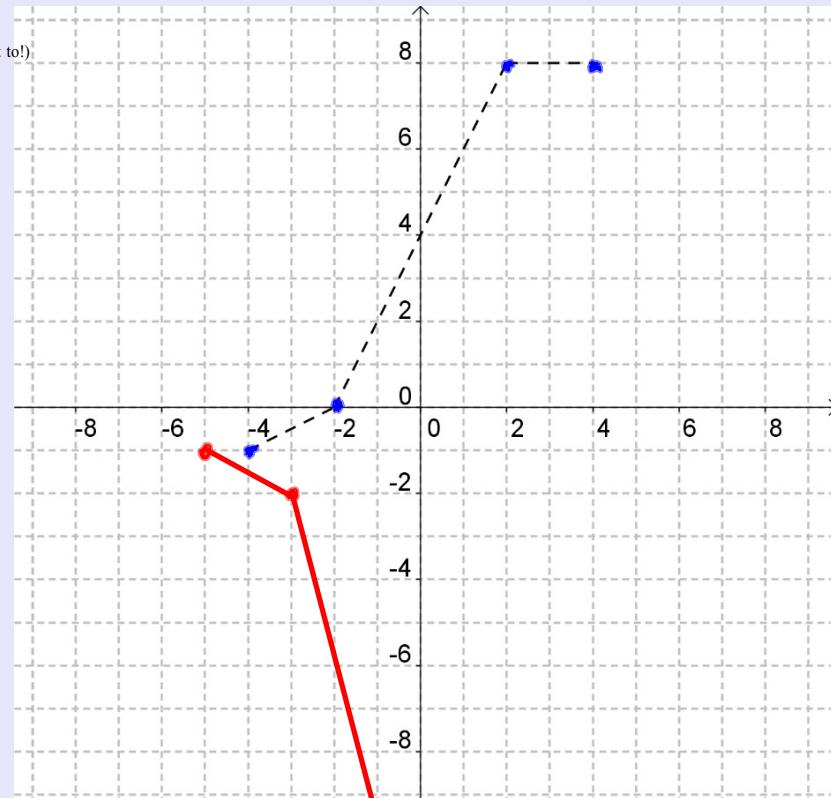
$$(-4, 1) \quad (-2, 0)$$

$\downarrow$  ②     $\downarrow$

$$(-5, 1) \quad (-3, 0)$$

$\downarrow$      $\downarrow$

$$(-5, -1) \quad (-3, -2)$$



$$(2, 8) \rightarrow (2, -8) \rightarrow (1, -8) \rightarrow (1, -10)$$

$$(4, 8) \rightarrow (4, -8) \rightarrow (3, -8) \rightarrow (3, -10)$$

Ex: The graph on the right shows the function  $y = f(x)$   
 $y = 2f(x) - 1$ , describe the transformations and  
apply them to the function.

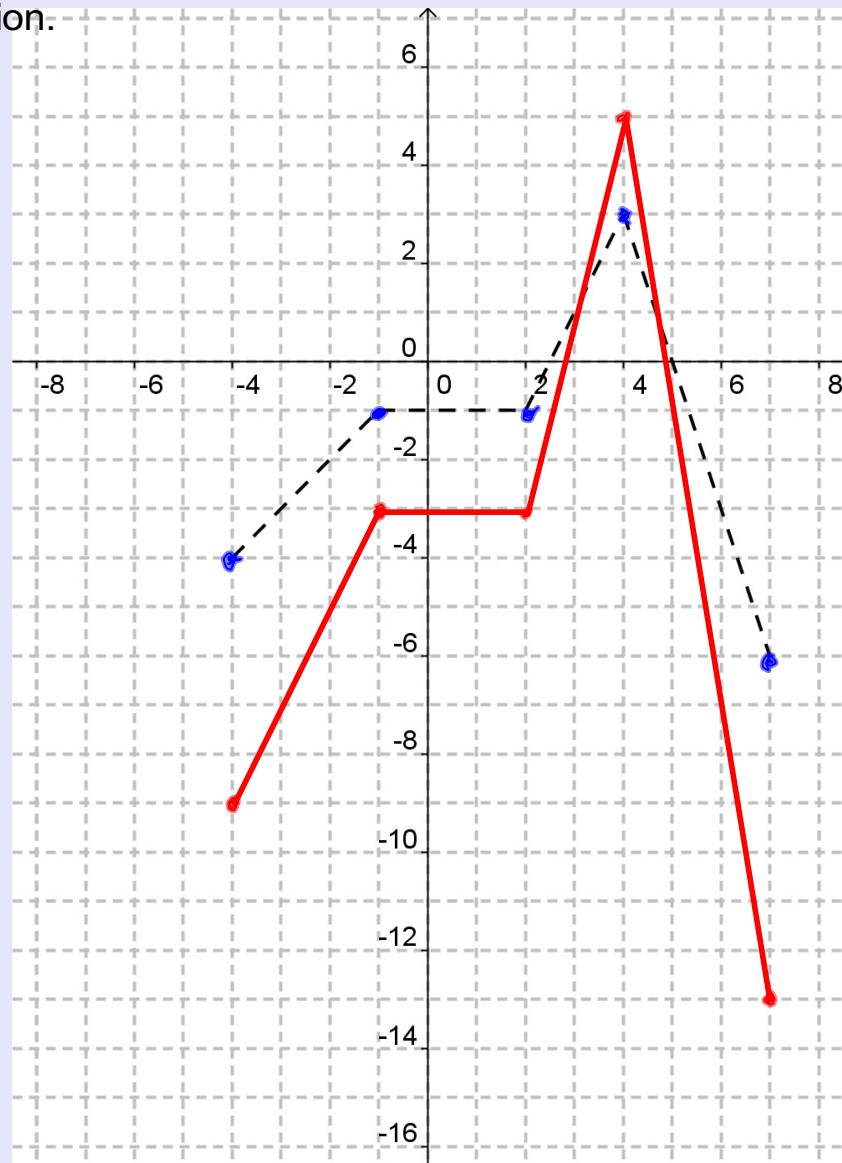
(Use a table of values with the key points, if you want to!)

① v. stretch by 2  
 $y \times 2$

② down by 1.

$$x_2 = x_1$$

$$y_2 = 2y_1 - 1$$



Ex: The graph on the right shows the function  $y = -0.5f(x+2)$ . Given  $f(x) = \sqrt{16 - x^2}$ , describe the transformations and apply them to the function.

(Use a table of values with the key points, if you want to!)

① v. reflect

② v. compress by 2

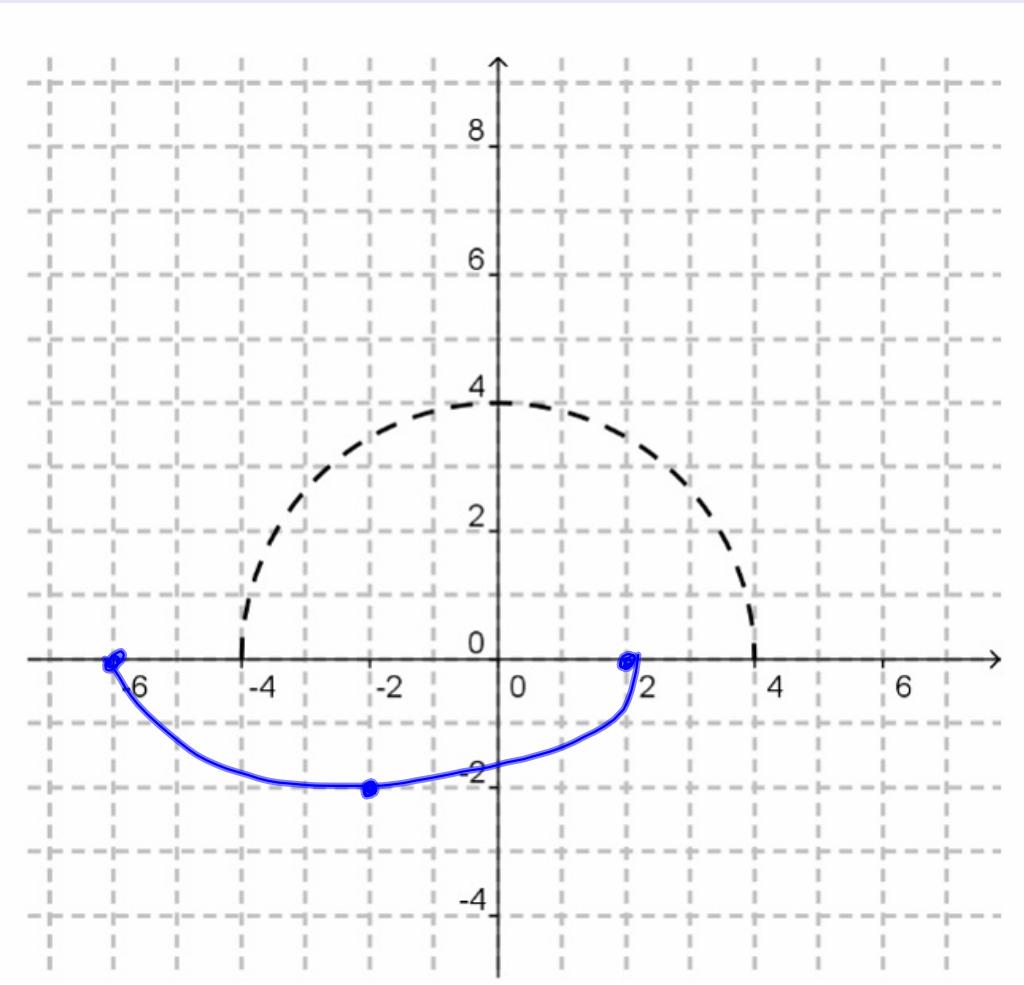
$$y \div 2 \text{ or } y \times 0.5$$

③ h. shift left by 2

$$(-4, 0) \rightarrow (-4, 0) \rightarrow (-4, 0) \rightarrow (-6, 0)$$

$$(0, 4) \rightarrow (0, -4) \rightarrow (0, -2) \rightarrow (-2, -2)$$

$$(4, 0) \rightarrow (4, 0) \rightarrow (4, 0) \rightarrow (2, 0)$$



## Assigned Work:

p.241 # 8b, 9b, 5abd, 4abce