

Transformations of Functions

Recall: In Gr.10, we explored transformations of a quadratic relation. For consistency, we use vertex form.

$$y = \underline{a}(x - h)^2 + k$$

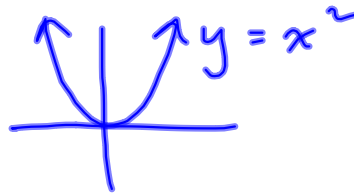
What are the possible transformations?

(read from left to right)

- ① a : $v.$ reflection?
vertical scaling $\begin{cases} \rightarrow \text{stretch} \\ \rightarrow \text{compression} \end{cases}$
- ② h : x_{vertex} horizontal shift left/right
- ③ k : y_{vertex} vertical shift up/down

$$y = a(x - h)^2 + k$$

1&2
3
4

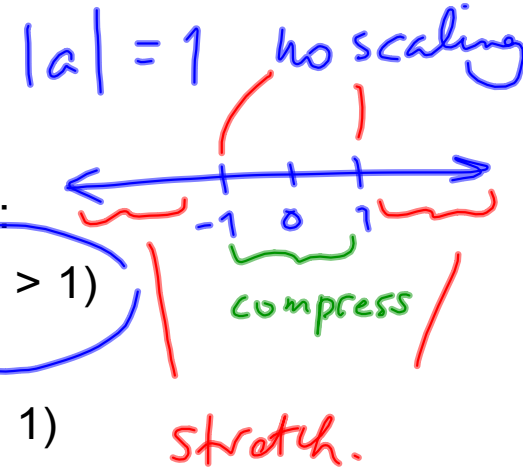


Summary of possible transformations (in order):

1. Vertical Reflection: $a < 0$

2. Vertical Scaling (stretch or compress):

- stretch when $a < -1$ or $a > 1$ (or $|a| > 1$)
- parabola is thinner
- compress when $-1 < a < 1$ (or $|a| < 1$)
- parabola is wider



3. Horizontal Translation (shift left or right) by h

4. Vertical Translation (shift up or down) by k

Using function notation,

$$y = x^2 \quad \text{becomes} \quad y = f(x)$$

$$y = a(x-h)^2 + k \quad \text{becomes} \quad y = af(x-h) + k$$

The transformations produced by a, h, and k can be applied to any function.

Start with:

$$f(x) = x^2$$

[xa]

$$af(x) = ax^2$$

$$af(x-h) = a(x-h)^2$$

$$af(x-h) + k = a(x-h)^2 + k$$

Ex.1 State the transformations, in the correct order,

$y = f(x)$ has undergone to obtain $y = f(x+1) + 3$

→ ① ②

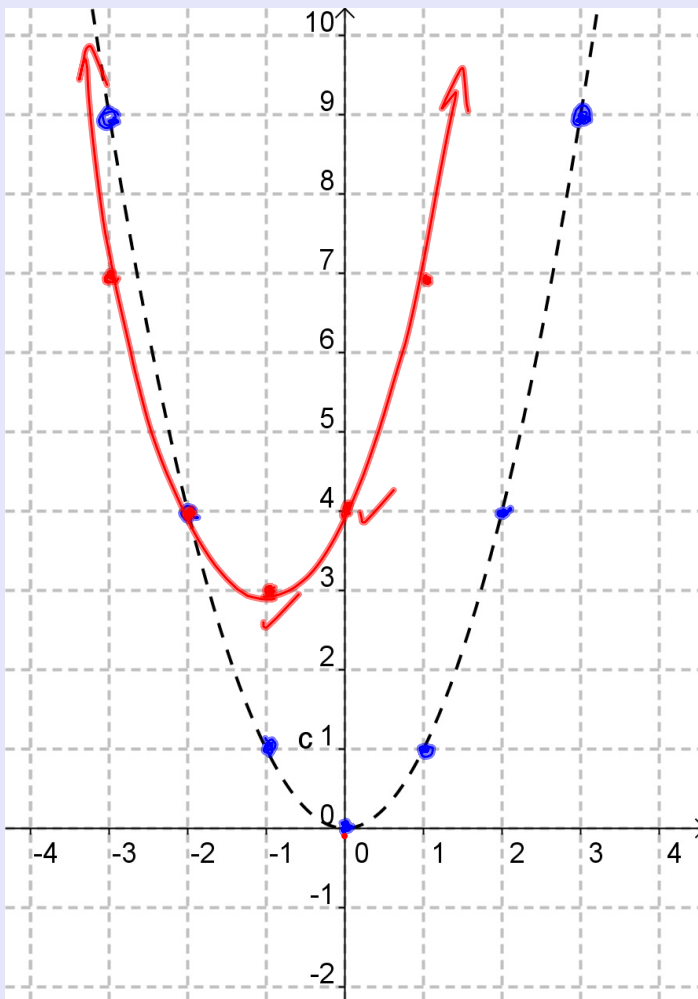
① h. shift left by 1

② v. shift up by 3

Using the previous example, what if the parent function is a quadratic? Then $f(x) = x^2$.

Sketch the transformed parabola on the grid below.

(Use a table of values if you want to)



$$y = f(x+1) + 3$$

① ②

① left 1

② up 3

$$(0,0) \xrightarrow{\textcircled{1}} (-1,0) \xrightarrow{\textcircled{2}} (-1,3)$$

x y

$$(1,1) \longrightarrow (0,4)$$

What if the parent function is a set of ordered pairs?

$$f(x) = \left\{ \underset{x}{(1, 2)}, \underset{x}{(2, -3)}, \underset{x}{(-5, 7)} \right\}$$

what would $f(x+1)+3$ become?

(Use a table of values and/or graph if you want to!)

① left by 1 (x)

② up by 3 (y)

$$(1, 2) \xrightarrow{\textcircled{1}} (0, 2) \xrightarrow{\textcircled{2}} (0, 5)$$

$$(2, -3) \rightarrow (1, -3) \rightarrow (1, 0)$$

$$(-5, 7) \rightarrow (-6, 7) \rightarrow (-6, 10)$$

$$f(x+1)+3 = \left\{ (0, 5), (1, 0), (-6, 10) \right\}$$

What if the parent function is the square root function?

Then $f(x) = \sqrt{x}$.

What would $f(x+1) + 3$ become?

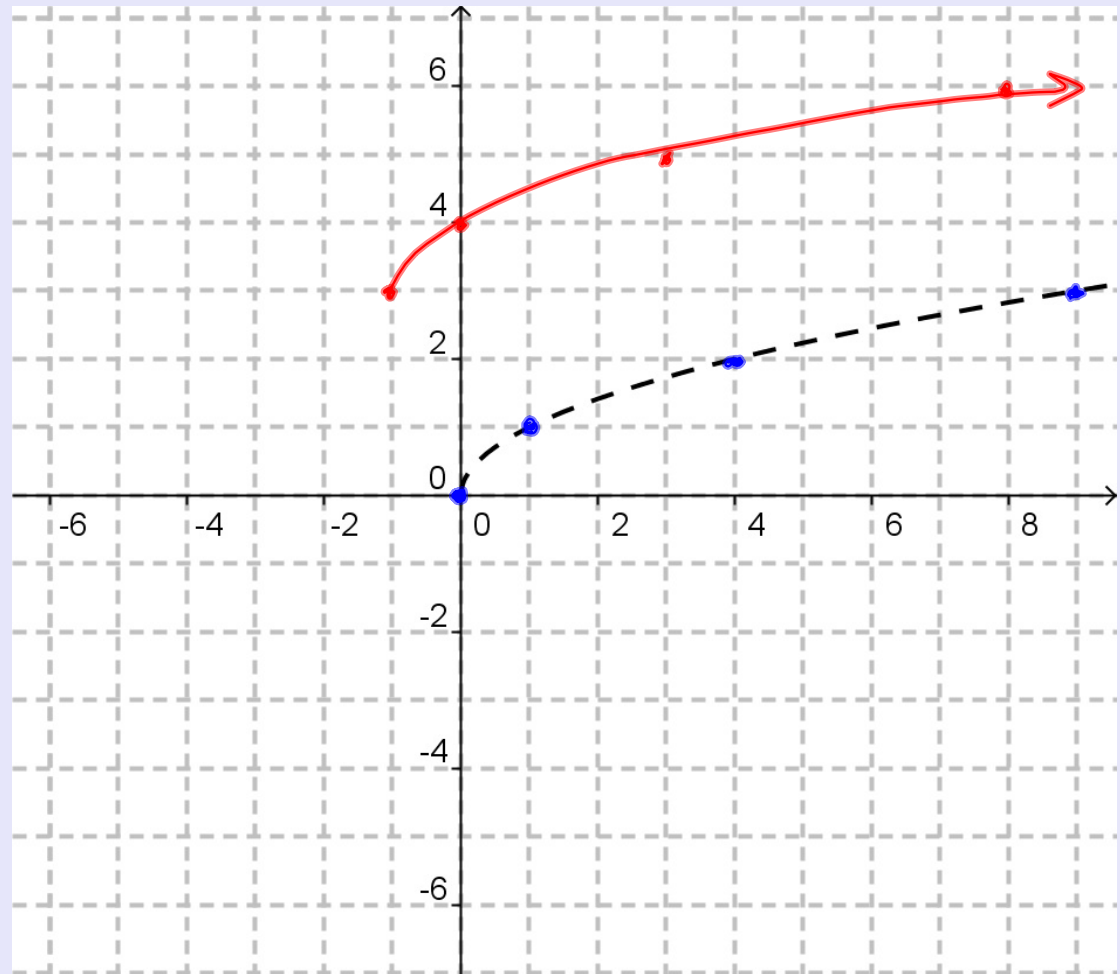
(Use a table of values if you want to)

$$(0, 0) \rightarrow (-1, 3)$$

$$(1, 1) \rightarrow (0, 4)$$

$$(4, 2) \rightarrow (3, 5)$$

$$(9, 3) \rightarrow (8, 6)$$



What if the parent function is the reciprocal function?

$$\text{Then } f(x) = \frac{1}{x} .$$

What would $f(x+1)+3$ become?

(Use a table of values if you want to)

$$(1, 1)$$

$$(-1, -1)$$

$$(2, 0.5)$$

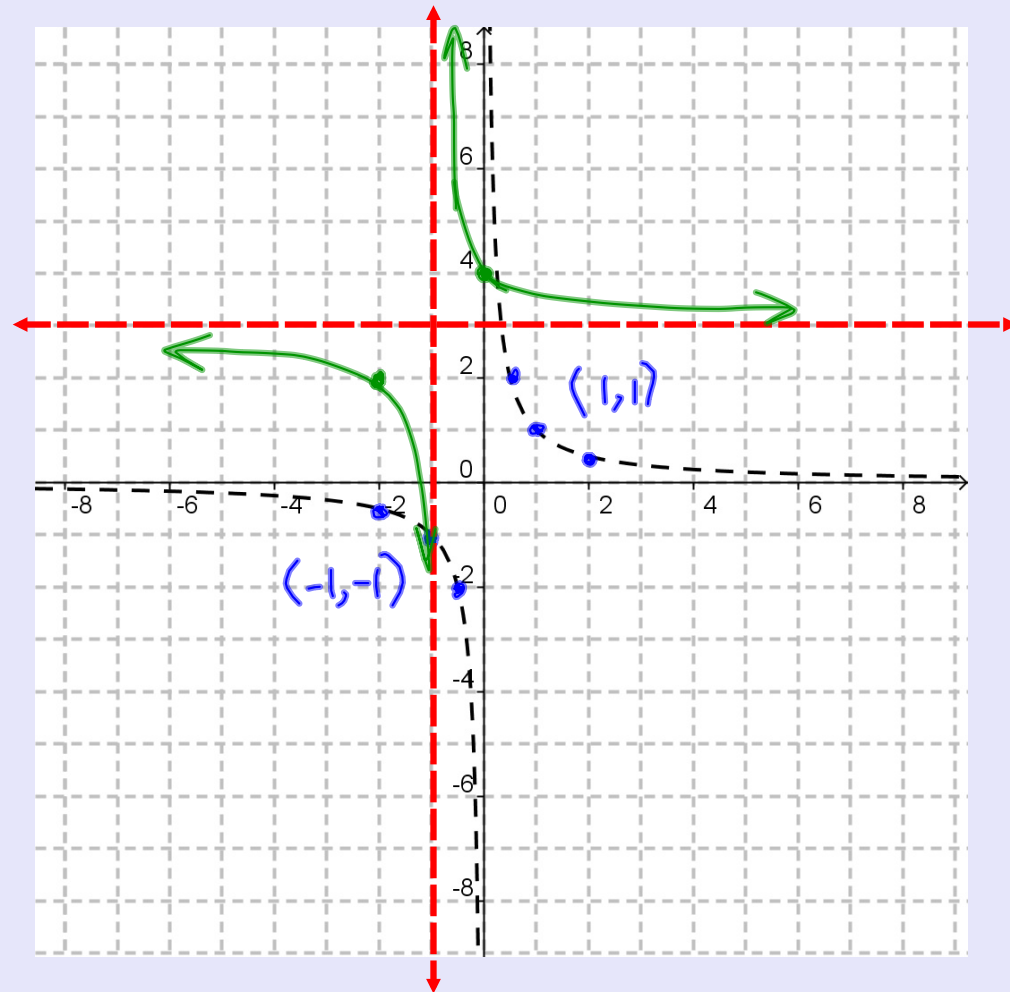
$$(0.5, 2)$$

$$(-2, -0.5)$$

$$(-0.5, -2)$$

$$\text{VA : } x = 0 \rightarrow x = -1$$

$$\text{HA : } y = 0 \rightarrow y = 3$$



$$y = 2\sqrt{x} + 3$$

describe the transformations and apply them to the square root function.

(Use a table of values and/or graph if you want to!)

$$y = 2\sqrt{x} + 3$$

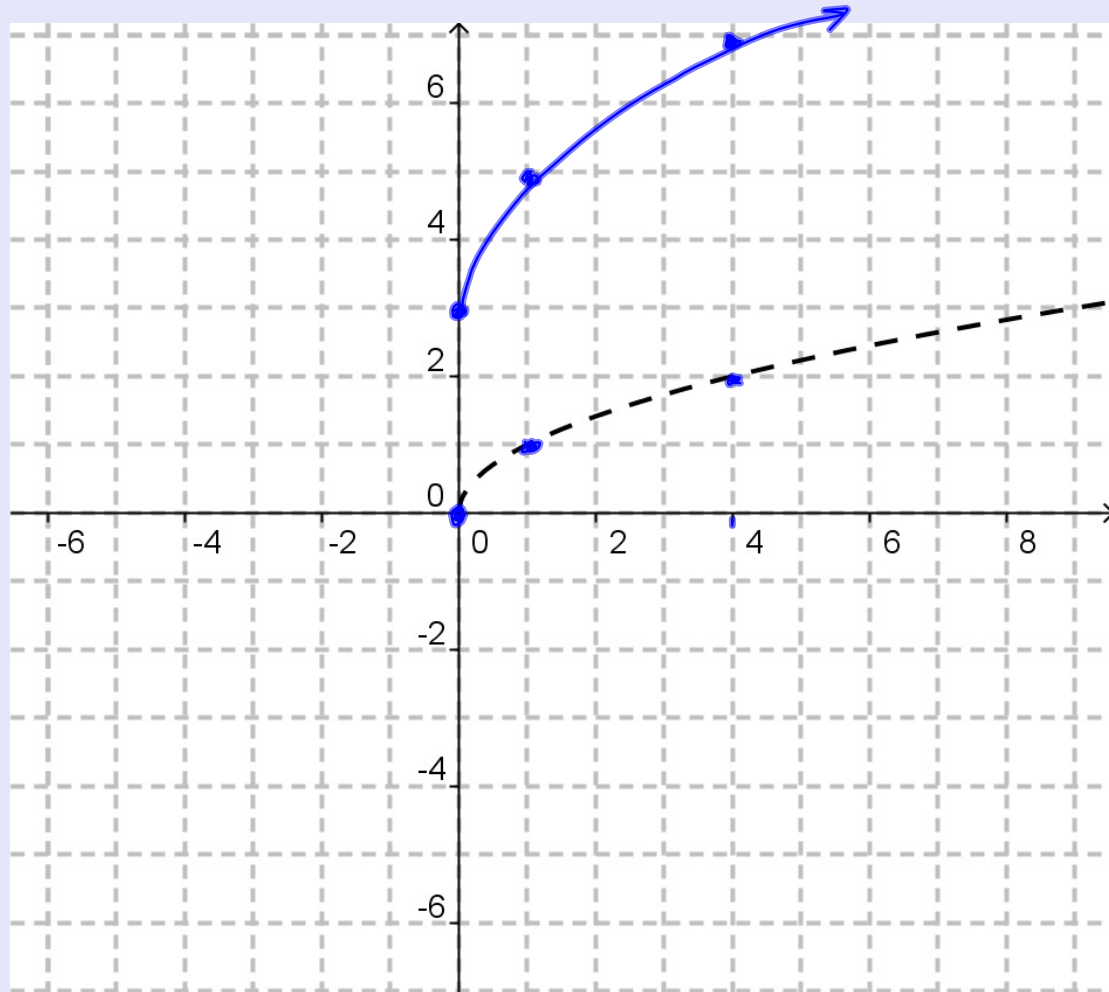
① ②

① $a = 2$
v. stretch by 2
 $y \times 2$

② v. shift by 3
 $y + 3$

$(0,0) \rightarrow (0,0) \rightarrow (0,3)$
①

$(1,1) \rightarrow (1,2) \rightarrow (1,5)$



Ex. 4 Given $y = 0.4f(x-1)$, describe the transformations and apply them to the reciprocal function.

(Use a table of values and/or graph if you want to!)

$$y = 0.4f(x-1)$$

(1) (2)

(1) : v. scaling by 0.4

$$y \times \frac{2}{5}$$

OR
 $\frac{4}{10} = \frac{2}{5}$

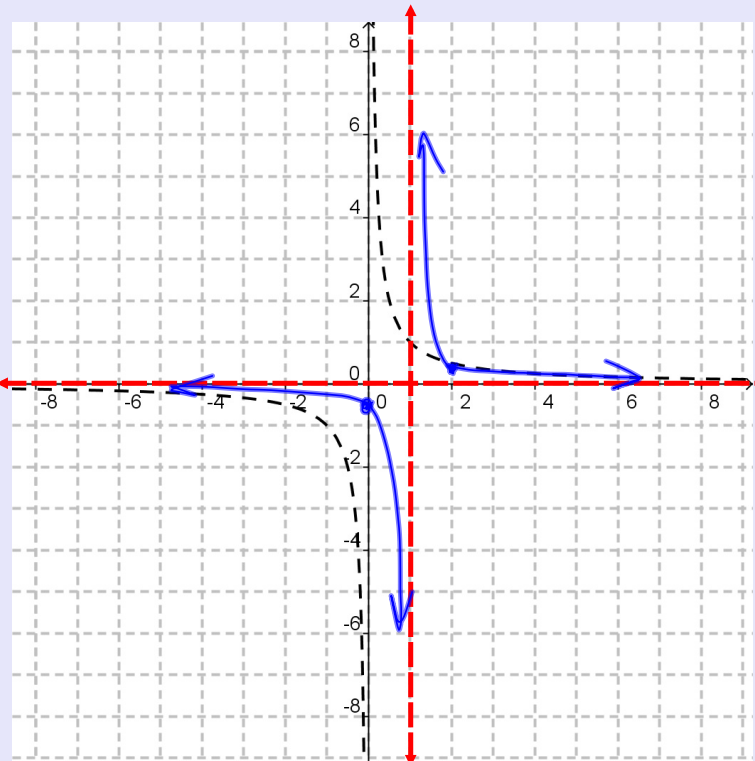
(c) v. compress by $\frac{5}{2}$ (>1)

$$y \div \frac{5}{2}$$

(c) v. compress by $\frac{2}{5}$

$$y \times \frac{2}{5}$$

(2) right by 1



$$VA : x = 0 \xrightarrow{(1)} x = 0 \xrightarrow{(2)} x = 1$$

$$HA : y = 0 \rightarrow y = 0 \rightarrow y = 0$$

$$(1, 1) \rightarrow (1, \frac{2}{5}) \rightarrow (2, \frac{2}{5})$$

(2, 0.4)

$$(-1, -1) \rightarrow (-1, -\frac{2}{5}) \rightarrow (0, -0.4)$$

Ex: The graph on the right shows the function $y = f(x)$.

~~Given~~
 $y = -f(x+1) - 2$
 , describe the transformations and apply them to the function

(Use a table of values with the key points, if you want to!)

$y = -f(x+1) - 2$
 ① ② ③

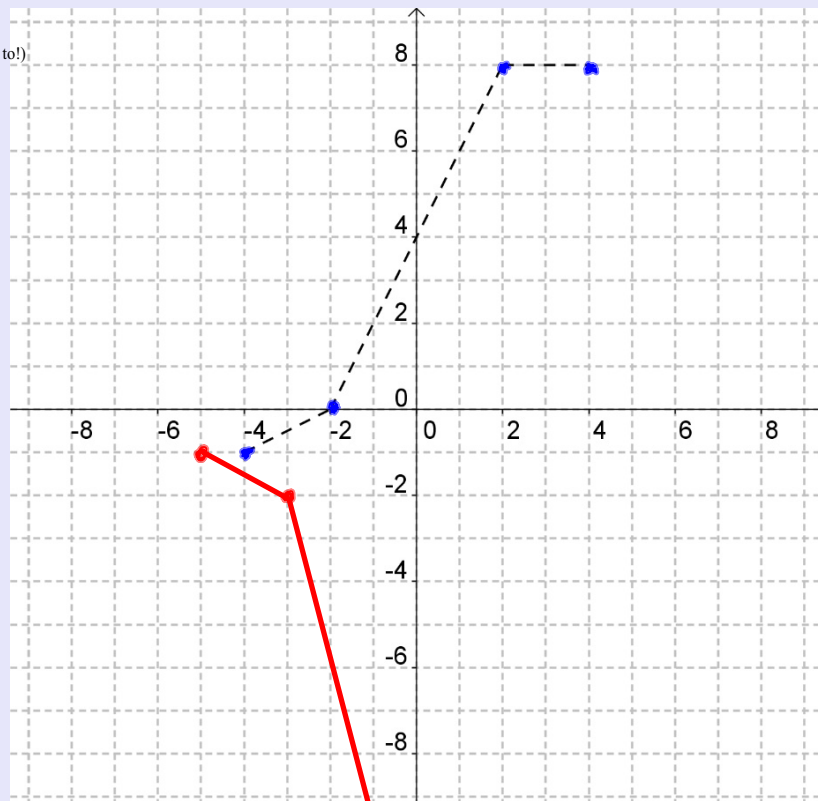
- ① : v.reflect
- ② : shift left by 1
- ③ : down by 2

$(-4, -1)$	$(-2, 0)$
↓ ①	↓

$(-4, 1)$	$(-2, 0)$
↓ ②	↓

$(-5, 1)$	$(-3, 0)$
↓	↓

$(-5, -1)$	$(-3, -2)$
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$(2, 8) \rightarrow (2, -8) \rightarrow (1, -8) \rightarrow (1, -10)$

$(4, 8) \rightarrow (4, -8) \rightarrow (3, -8) \rightarrow (3, -10)$

Ex: The graph on the right shows the function $y = f(x)$, describe the transformations and apply them to the function.

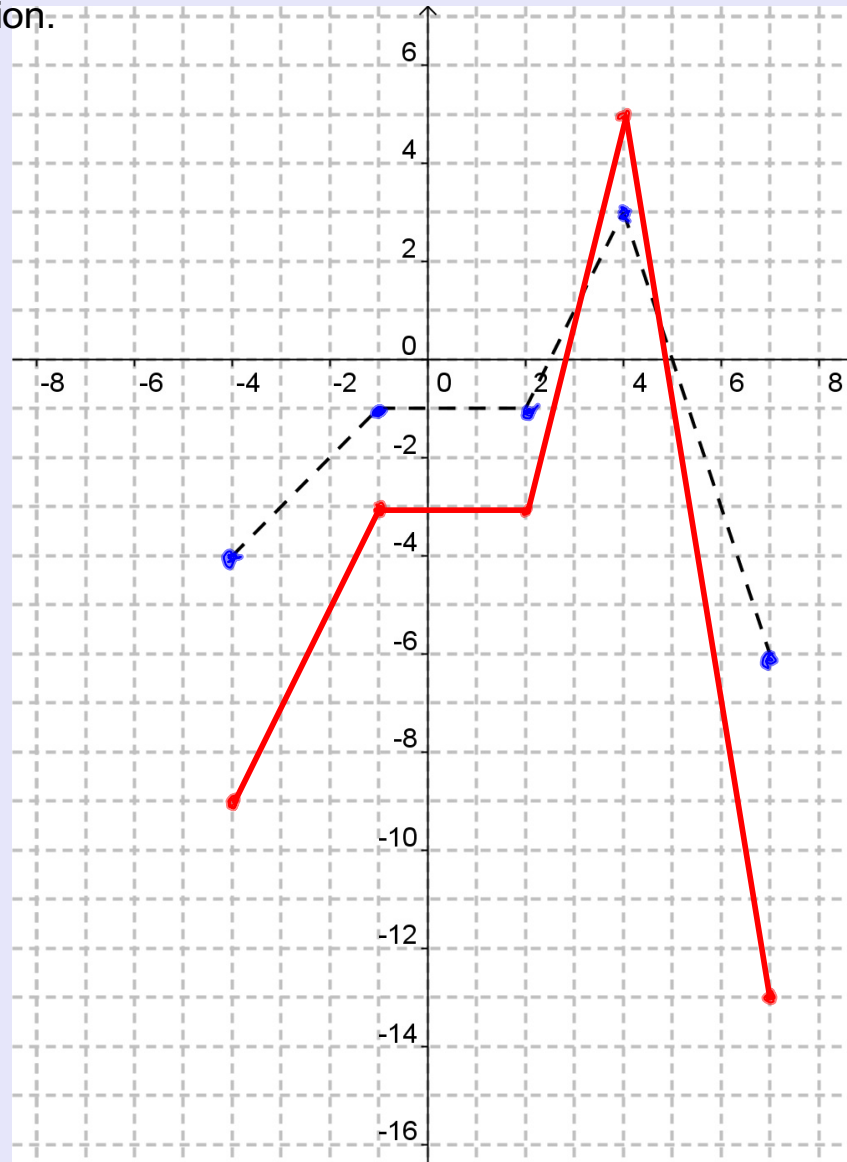
(Use a table of values with the key points, if you want to!)

① v. stretch by 2
 $y \times 2$

② down by 1.

$$x_2 = x_1$$

$$y_2 = 2y_1 - 1$$



Ex: The graph on the right shows the function $f(x) = \sqrt{16 - x^2}$.

Given $y = -0.5f(x+2)$, describe the transformations and apply them to the function.

(Use a table of values with the key points, if you want to!)

① v. reflect

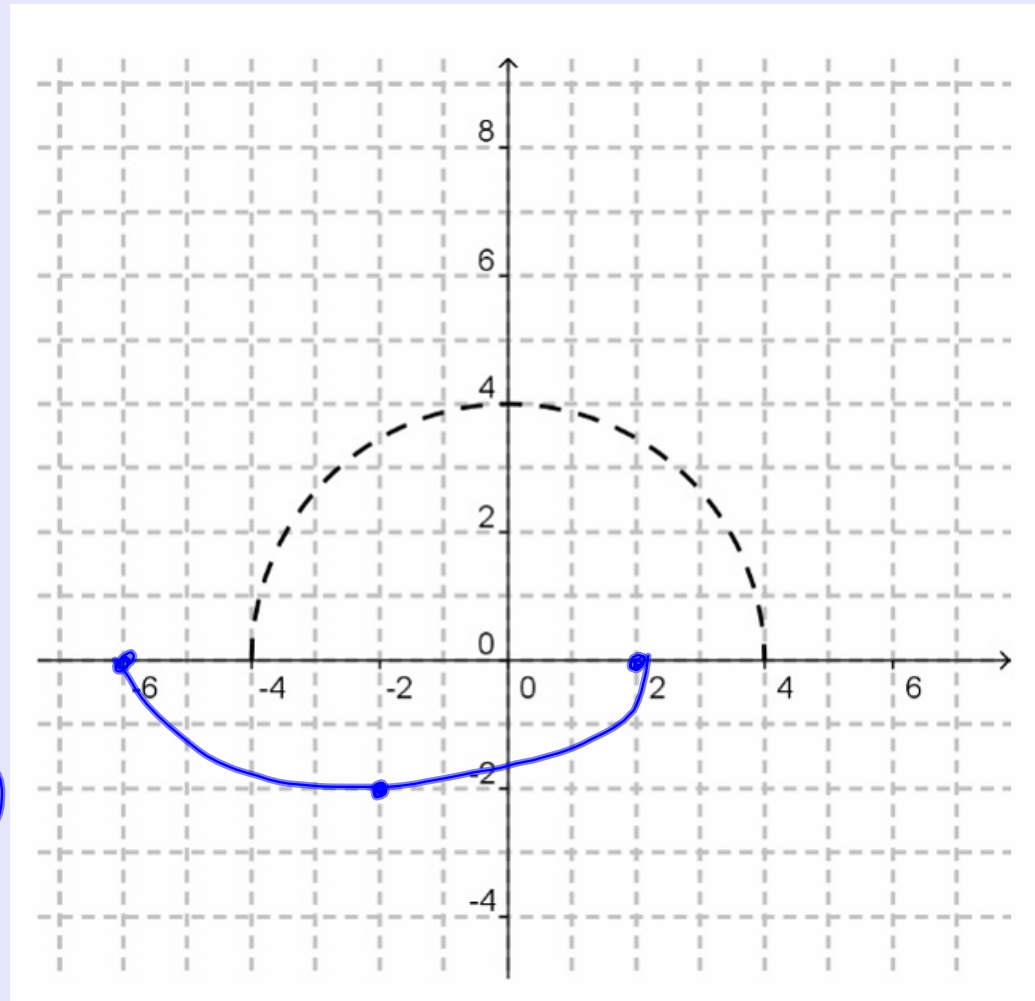
② v. compress by 2
 $y \div 2$ or $y \times 0.5$

③ h. shift left by 2

$(-4, 0) \rightarrow (-4, 0) \rightarrow (-4, 0) \rightarrow (-6, 0)$

$(0, 4) \rightarrow (0, -4) \rightarrow (0, -2) \rightarrow (-2, -2)$

$(4, 0) \rightarrow (4, 0) \rightarrow (4, 0) \rightarrow (2, 0)$



Assigned Work:

p.241 # 8b, 9b, 5abd, 4abce