

Transformations of Functions

see handout

Recall: In Gr.10, we explored transformations of a quadratic relation. For consistency, we use vertex form.

$$y = a(x - h)^2 + k$$

What are the possible transformations?
(read from left to right)

a: direction of opening  
Vertical reflection for $a < 0$

a: $a > 1$ or $a < -1$, v. stretch

 $|a| > 1$

a: $-1 < a < 1$, v. compression $|a| < 1$
 h: horizontal shift
 k: vertical shift

Feb 27-10:44 AM

$$y = a(x - h)^2 + k$$

1 & 2 3 4

Summary of possible transformations (in order):

1. Vertical Reflection: $a < 0$
2. Vertical Scaling (stretch or compress):
 - stretch when $a < -1$ or $a > 1$ (or $|a| > 1$)
 - parabola is thinner
 - compress when $-1 < a < 1$ (or $|a| < 1$)
 - parabola is wider
3. Horizontal Translation (shift left or right) by h
4. Vertical Translation (shift up or down) by k

Feb 27-10:48 AM

Using function notation,

$$y = x^2 \text{ becomes } y = f(x)$$

$$\text{so } y = a(x-h)^2 + k \text{ becomes } y = af(x-h) + k$$

The transformations produced by a, h, and k can be applied to any function.

Start with: $f(x) = x^2$

$$af(x) = ax^2$$

$$af(x-h) = a(x-h)^2$$

$$af(x-h) + k = a(x-h)^2 + k$$

Feb 27-10:54 AM

Ex.1 State the transformations, in the correct order, that $y = f(x)$ has undergone to obtain $y = f(x+1)+3$

① ②

①: h. shift left by 1 $y = (x+1)^2 + 3$

②: v. shift up by 3 $V(-1, 3)$

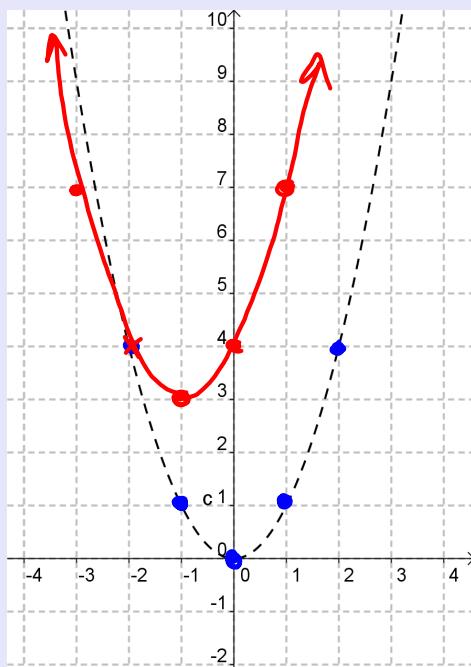
$$y = x^2
V(0, 0)$$

Feb 27-10:54 AM

Using the previous example, what if the parent function is a quadratic? Then $f(x) = x^2$.

Sketch the transformed parabola on the grid below.

(Use a table of values if you want to)



$$y = f(x+1)+3$$

$$f(x) = x^2$$

$$y = (x+1)^2 + 3$$

$$V(-1, 3)$$

$$(0, 0) \xrightarrow{x-1} (-1, 0) \xrightarrow{y+3} (-1, 3)$$

$$(1, 1) \rightarrow (0, 1) \rightarrow (0, 4)$$

Mar 2-6:09 AM

What if the parent function is a set of ordered pairs?

$$\text{If } f(x) = \{(1, 2), (2, -3), (-5, 7)\}$$

what would $y = f(x+1)+3$ become?

(Use a table of values and/or graph if you want to!)

$$\textcircled{1}: \text{Left by } 1 \quad x-1$$

$$\textcircled{2}: \text{up by } 3 \quad y+3$$

$$\begin{array}{c} \xrightarrow{\text{old}} \\ (1, 2) \end{array} \xrightarrow{\textcircled{1}} (0, 2) \xrightarrow{\textcircled{2}} \begin{array}{c} \xrightarrow{\text{new}} \\ (0, 5) \end{array}$$

$$(2, -3) \xrightarrow{\textcircled{1}, \textcircled{2}} (1, 0)$$

$$(-5, 7) \xrightarrow{\textcircled{1}, \textcircled{2}} (-6, 10)$$

$$\therefore \text{transformed function is } \{(0, 5), (1, 0), (-6, 10)\}$$

Mar 2-6:11 AM

What if the parent function is the square root function?

Then $f(x) = \sqrt{x}$.

What would $y = f(x+1) + 3$ become?

(Use a table of values if you want to)

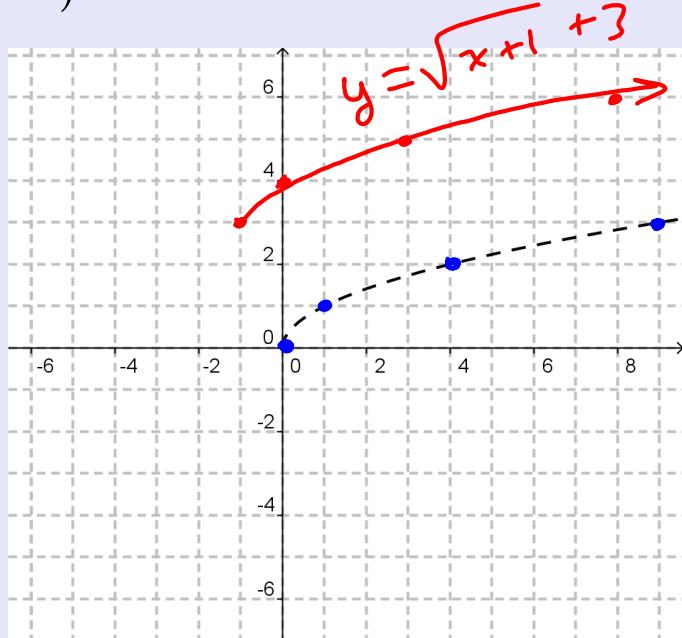
$$(0,0) \rightarrow (-1,3)$$

$$(1,1) \rightarrow (0,4)$$

$$(4,2) \rightarrow (3,5)$$

$$(9,3) \rightarrow (8,6)$$

$$y = \sqrt{x+1} + 3$$



Mar 2-6:13 AM

What if the parent function is the reciprocal function?

Then $f(x) = \frac{1}{x}$.

What would $y = f(x+1) + 3$ become?

(Use a table of values if you want to)

① ②

①: left by 1 $x-1$

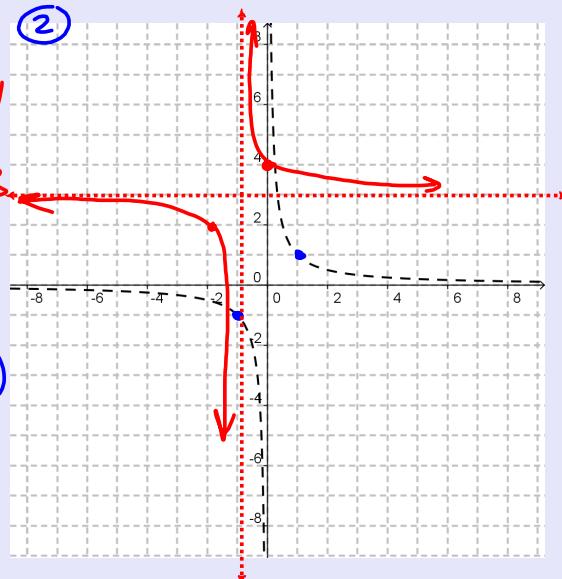
②: up by 3 $y+3$

$$(1,1) \rightarrow (0,4)$$

$$(-1,-1) \rightarrow (-2,2)$$

$$\text{VA: } x=0 \rightarrow x=-1$$

$$\text{HA: } y=0 \rightarrow y=3$$



$$D = \{x \in \mathbb{R} \mid x \neq -1\} \quad R = \{y \in \mathbb{R} \mid y \neq 3\}$$

Mar 2-6:15 AM

Ex: Given $y = 2f(x) + 3$ describe the transformations and apply them to the square root function.

(Use a table of values and/or graph if you want to!)

$$y = 2x^2 + 3$$

$$V(0, 3)$$

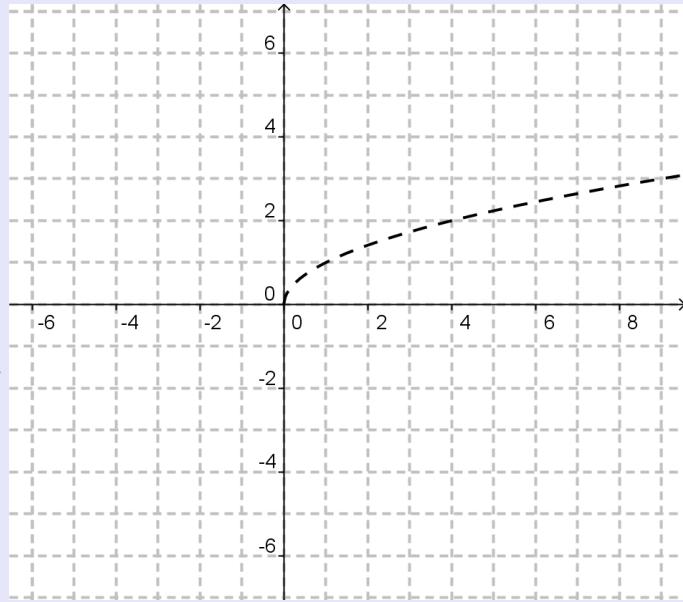
$$y = 2(x-0)^2 + 3$$

Step pattern

$$1, 3, 5, \dots$$



$$2, 6, 10, \dots$$



Mar 2-6:17 AM

Ex: Given $y = 2f(x) + 3$ describe the transformations and apply them to the square root function.

(Use a table of values and/or graph if you want to!)

$$y = 2f(x) + 3$$

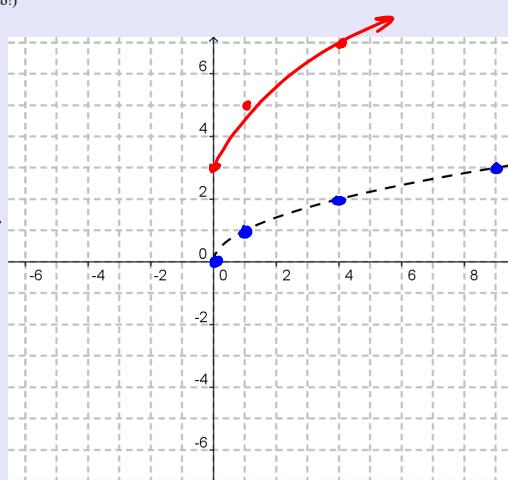
① ②

① Vertical stretch by 2

$$2y$$

∴ V. shift up by 3

$$y+3$$



$$(0, 0) \xrightarrow{①} (0, 0) \xrightarrow{②} (0, 3)$$

$$(1, 1) \rightarrow (1, 2) \rightarrow (1, 5)$$

$$(4, 2) \rightarrow (4, 4) \rightarrow (4, 7)$$

Mar 2-6:17 AM

x	$f(x) = x^2$	$y = 2f(x) = 2x^2$
-3	9	18
-2	4	8
-1	1	2
0	0	0
1	1	2
2	4	8
3	9	18

x y

Mar 1-1:23 PM

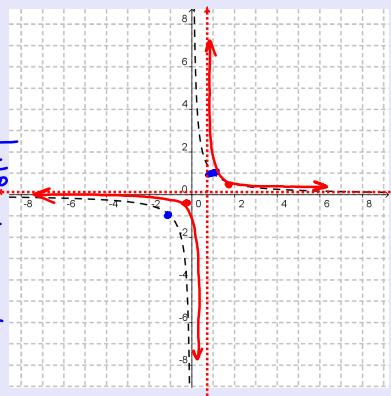
Ex: Given $y = 0.4f(x-1)$, describe the transformations and apply them to the reciprocal function.

(Use a table of values and/or graph if you want to!)

① Vertical compression
by $\frac{1}{0.4} = \frac{1}{\frac{2}{5}} = \frac{5}{2}$

$$\begin{aligned} y &\div 2.5 \\ &= \frac{10}{4} \\ &= \frac{5}{2} \\ &= 2.5 \end{aligned}$$

V. compression
by 0.4 $y \times 0.4$



② right by 1, $x+1$

$$(1, 1) \xrightarrow[4 \times 0.4]{\textcircled{1}} (1, 0.4) \xrightarrow{x+1}{\textcircled{2}} (2, 0.4)$$

$$(-1, -1) \rightarrow (-1, -0.4) \rightarrow (0, -0.4)$$

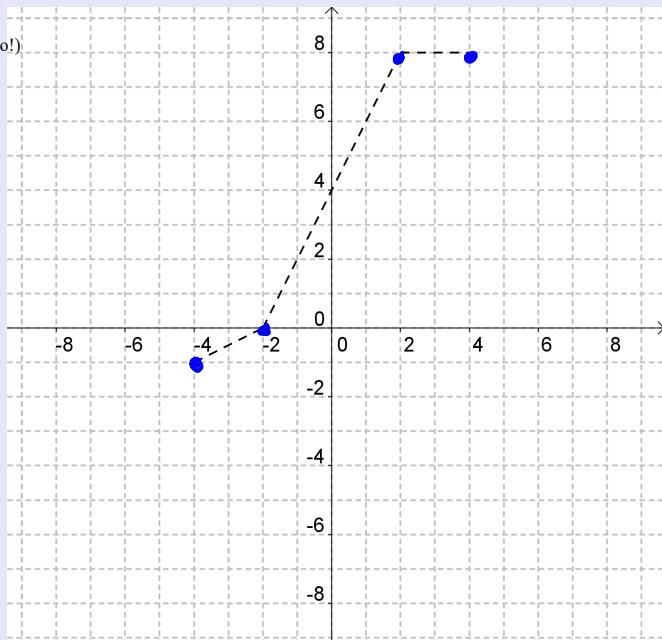
$$\text{VA: } x = 0 \xrightarrow{\textcircled{1}} x = 0 \rightarrow x = 1$$

$$\text{HA: } y = 0 \rightarrow y = 0 \rightarrow y = 0$$

Mar 2-6:18 AM

Ex: The graph on the right shows the function $y = f(x)$.
 Given $y = -f(x+1) + 2$, describe the transformations and apply them to the function

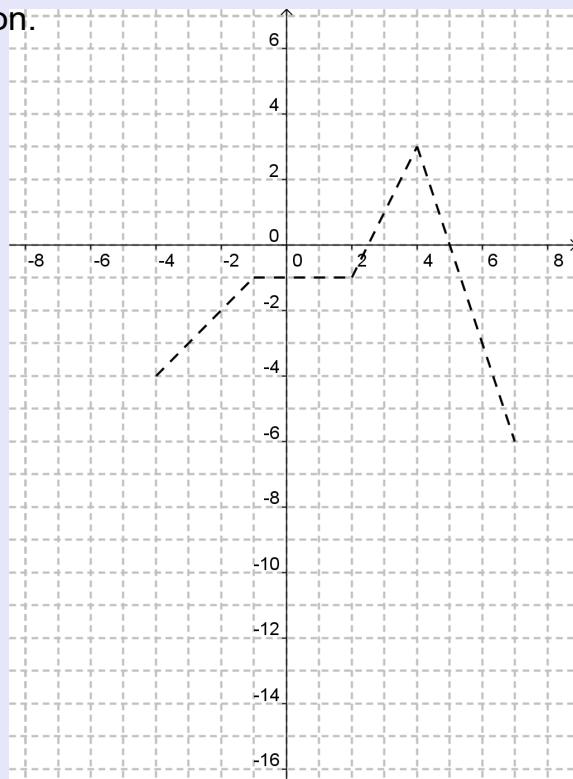
(Use a table of values with the key points, if you want to!)



Mar 2-6:20 AM

Ex: The graph on the right shows the function $y = f(x)$.
 Given $y = 2f(x) - 1$, describe the transformations and apply them to the function.

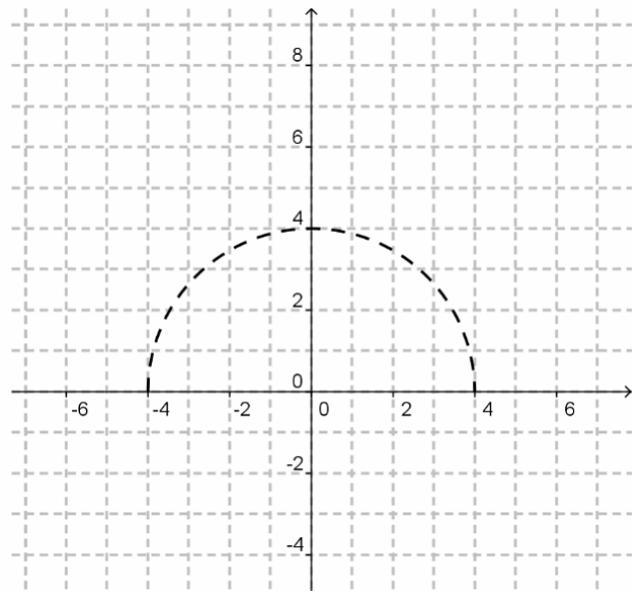
(Use a table of values with the key points, if you want to!)



Mar 2-6:22 AM

Ex: The graph on the right shows the function $f(x) = \sqrt{16 - x^2}$. Given $y = -0.5f(x+2)$, describe the transformations and apply them to the function.

(Use a table of values with the key points, if you want to!)

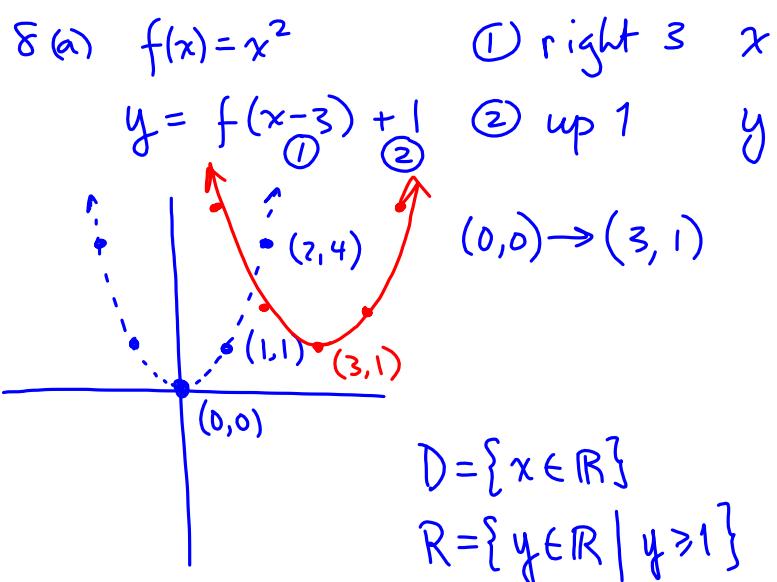


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Assigned Work:

p.241 # 8b, 9b, 5abd, 4abce

8a, 9a, 5ab, 4ab

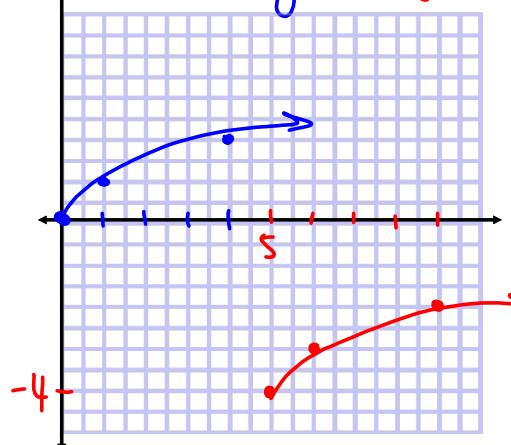


Mar 2-6:26 AM

$$9(a) \quad f(x) = \sqrt{x}$$

$$y = f(x-5) - 4 \quad \sqrt{x}, x \geq 0$$

- ① right by 5 $x+5$ $\sqrt{x-10}, x \geq 10$
 ② down by 4 $y-4$ $\sqrt{x+10}, x \geq -10$



$$(0,0) \rightarrow (5,-4)$$

$$(1,1) \rightarrow (6,-3)$$

$$(4,2) \rightarrow (9,-2)$$

$$D = \{x \in \mathbb{R} \mid x \geq 5\}$$

$$R = \{y \in \mathbb{R} \mid y \geq -4\}$$

Mar 1-12:44 PM

4 ab

$$(a) \quad y = f(x-4) + 2$$

① right by 4

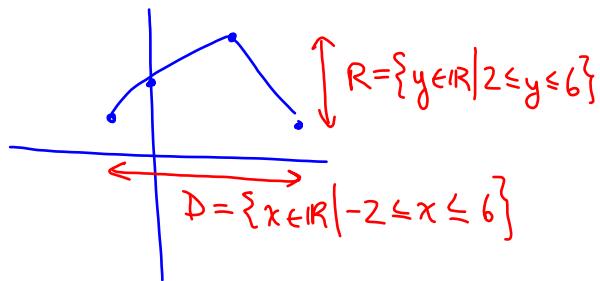
② up by 2

$$(-6,0) \rightarrow (-2,0) \rightarrow (-2,2)$$

$$(-4,2) \rightarrow (0,2) \rightarrow (0,4)$$

$$(0,4) \rightarrow (4,6)$$

$$(2,0) \rightarrow (6,2)$$



$$R = \{y \in \mathbb{R} \mid 2 \leq y \leq 6\}$$

$$D = \{x \in \mathbb{R} \mid -2 \leq x \leq 6\}$$

Mar 1-12:52 PM

Assigned Work:

p.241 # 8b, 9b, 5abd, 4abc

(8a), (9a), 5ab, 4ab

$$5(d) \quad y = 3f(x) - 2$$

① ②

① v. stretch by 3 $y \times 3$ ② down by 2 $y - 2$

$$(-2, 0) \xrightarrow{\textcircled{1}} (-2, 0) \xrightarrow{\textcircled{2}} (-2, -2)$$

$$(0, 5) \rightarrow (0, 15) \rightarrow (0, 13)$$

$$(2, -3) \xrightarrow{\hspace{2cm}} (2, -11)$$

$$(4, 0) \rightarrow (4, 0) \rightarrow (4, -2)$$

Mar 2-6:26 AM

$$4(c) \quad y = \frac{1}{2}f(x) - 3$$

① ②

① v. compression by 2 $y \div 2$

$$\frac{1}{2} \quad y \times \frac{1}{2}$$

② shift down by 3 $y - 3$

$$(-6, 0) \xrightarrow{\textcircled{1}} (-6, 0) \xrightarrow{\textcircled{2}} (-6, -3)$$

$$(-4, 2) \rightarrow (-4, 1) \rightarrow (-4, -2)$$

Mar 4-2:04 PM