# Determining Transformed Functions from Graphs

Tips for parabolas: $y = a(x - p)^2 + q$ $y = x^2$

1. The vertex of the parent function is at (0, 0). The value zero is not affected by scaling (a or k), only translations (p or q). The vertex will be at (p, q).

2. Parabolas can ignore the horizontal scaling, k, because there is an equivalent 'a' value.

k

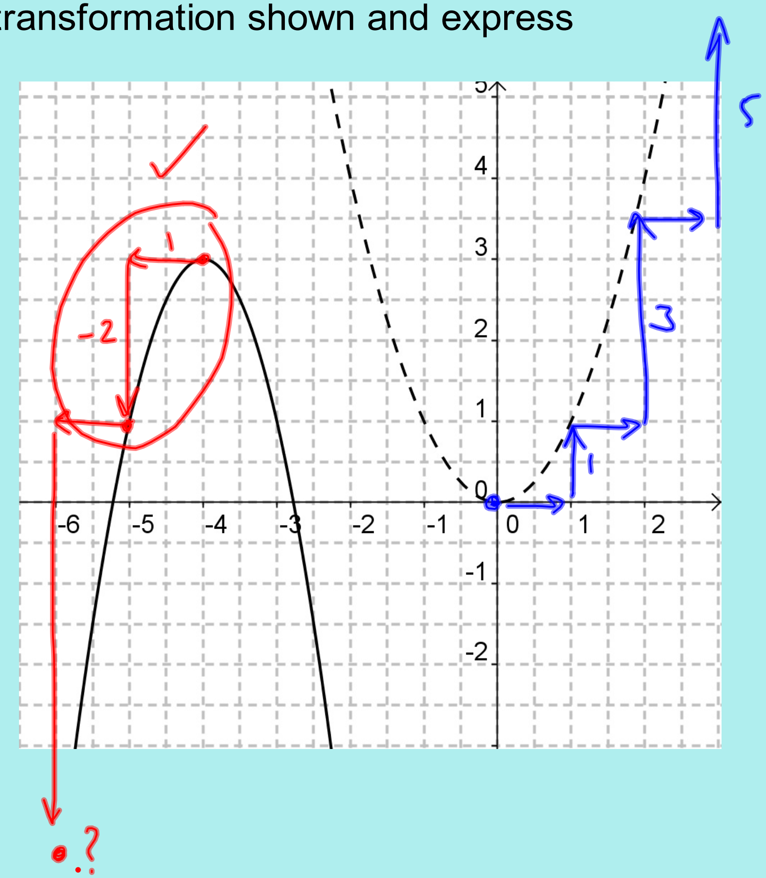
$$\boxed{\begin{aligned} y &= (3x)^2 \\ y &= 9x^2 \end{aligned}}$$

a

$y = x^2$

3. Use the step pattern (1, 3, 5, ...) from the vertex to determine the vertical scaling, 'a'.

Ex.1 Determine the transformation shown and express in function notation.



$(0,0) \to (-4, 3)$

$p = -4 \quad q = 3$

assume $k = 1$

$a < 0$

$a = -2$

step

$\{1, 3, 5\} \to \{-2, \underset{?}{-6}, \underset{?}{-10}\}$

$y = x^2$

$a = 1$

$$y = -2(x+4)^2 + 3$$

OR

$$y = -2f(x+4) + 3, \quad f(x) = x^2$$

Tips for radicals: $y = a\sqrt{k(x-p)} + q$

1. The parent function starts at (0, 0), just like a parabola. The value zero is not affected by scaling (a or k), only translations (p or q).

2. The sign of 'a' and 'k' are <u>both important</u> for reflections.

3. Use <u>one of</u> 'a' or 'k' for scaling. The horizontal scaling is more likely to give a "nice" (integer) value.

Ex.2 Determine the transformations shown and express in function notation.

$(0,0) \to (-2,-3)$

$p = -2 \quad q = -3$

h. stretched by 4

$x \to \frac{x}{k} \quad k = \frac{1}{4}$

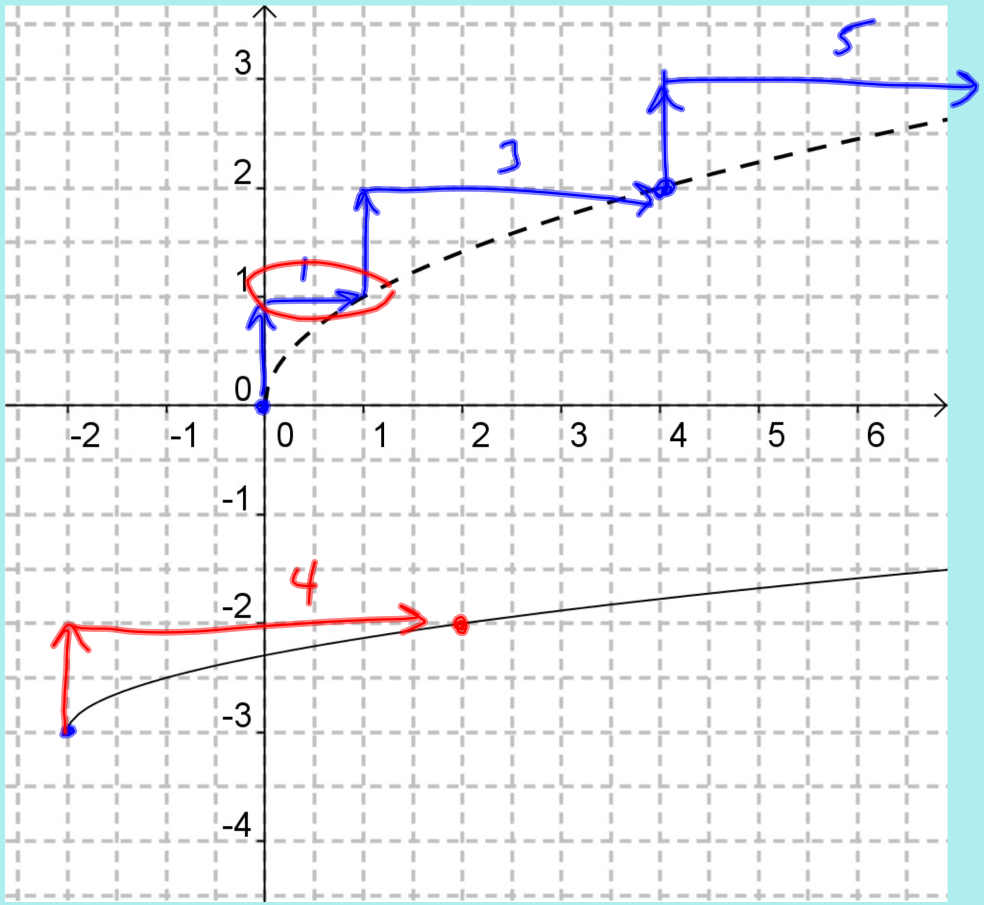
$k > 0 \quad a > 0$

no reflections

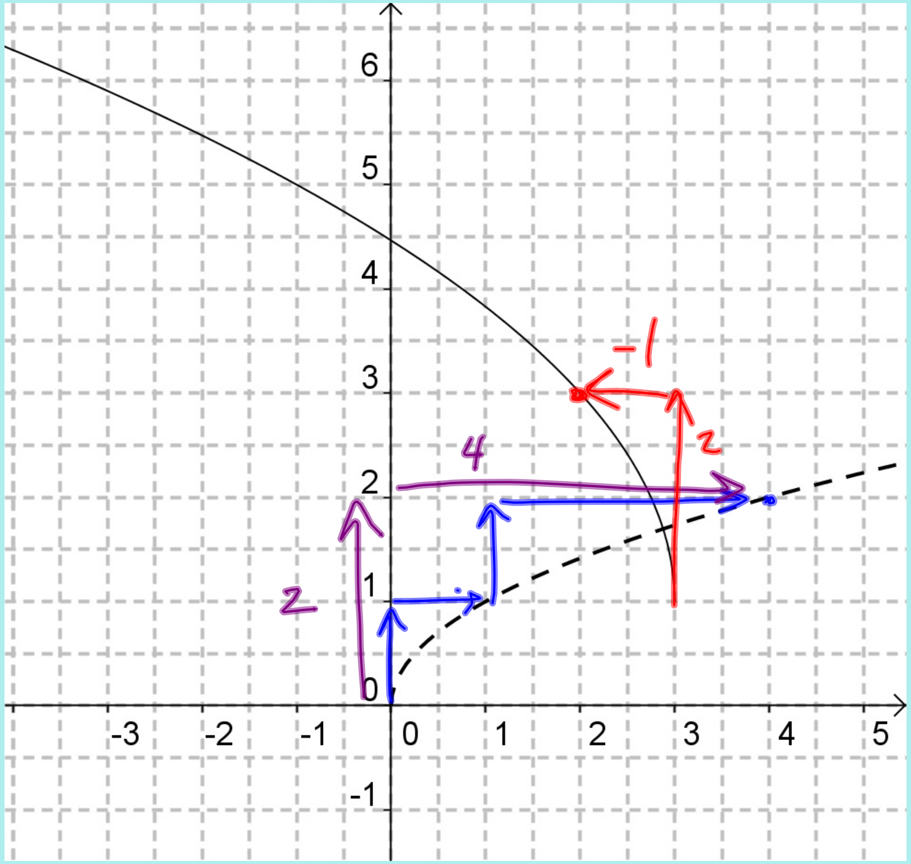
$y = \sqrt{\frac{1}{4}(x+2)} - 3$

or

$y = f\left[\frac{1}{4}(x+2)\right] - 3, \quad f(x) = \sqrt{x}$



Ex.3 Determine the transformations shown and express in function notation.




$(0,0) \rightarrow (3,1)$

$p = 3 \quad q = 1$

h. reflect : $\underline{\underline{k<0}}$

h. compress by 4 $[\div 4]$

$k = -4$

$y = \sqrt{-4(x-3)} + 1$

or

$y = f[-4(x-3)] + 1, \quad f(x) = \sqrt{x}$

Tips for rationals: $y = \frac{a}{k(x-p)} + q$

$y = \frac{1}{x}$

VA

1. The parent function has asymptotes at x=0 and y=0.

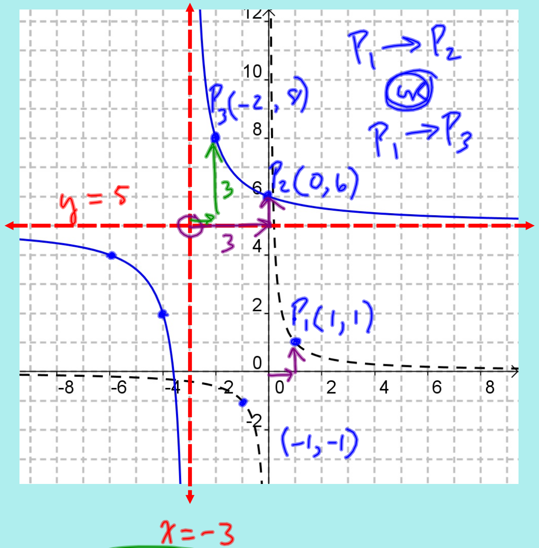
   The new asymptotes will be at x = p and y = q. HA

2. Use only one of 'a' or 'k' for scaling and reflection.

$$y = \frac{1}{3(x-2)} \Leftrightarrow y = \frac{1}{3} \times \frac{1}{x-2}$$

$k = 3$ $\qquad$ $a = \frac{1}{3}$

Ex.4 Determine the transformations shown and express in function notation.




$x=0 \to x=-3$

$p=-3$

$y=0 \to y=5$

$q=5$

① $P_1 \to P_2$

h. step of 1 $\to$ h. step of 3

h. stretch of 3

$k=\frac{1}{3}$

$y = 1f\left[\frac{1}{3}(x+3)\right]+5$, $f(x)=\frac{1}{x}$

$= \dfrac{1}{\frac{1}{3}(x+3)}+5$

$\dfrac{1}{\frac{1}{3}} = \frac{1}{1}\times\frac{3}{1}$

$=3$

$y = \dfrac{3}{x+3}+5$

② $P_1 \to P_3$

v. stretch by 3

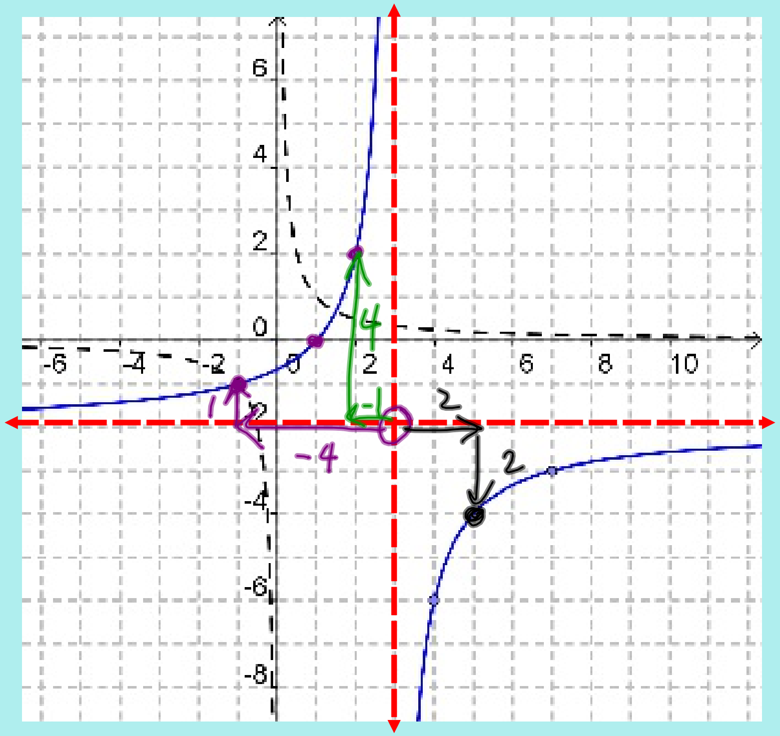
$a=3$

$y = 3f\left[1(x+3)\right]+5$

$= \frac{3}{1}\left(\dfrac{1}{x+3}\right)+5$

$y = \dfrac{3}{x+3}+5$

Ex.5 Determine the transformations shown and express in function notation.




$p = 3$ $q = -2$

$a < 0$ or $k < 0$

h. stretch by 4

$k = \frac{-1}{4}$

OR

v. stretch by 4

$a = -4$

OR

h. stretch by 2 AND v. stretch by 2

$k = \frac{-1}{2}$ $a = 2$

$y = -4f(x-3) - 2$

$y = \frac{-4}{x-3} - 2$

Tips for unknown functions, or collections of points:

1. Look for zeroes, since they are only affected by horizontal and vertical shifts.

$$(0,3) \rightarrow (p,?)$$

$$(0,0) \rightarrow (p,q)$$

$$(7,0) \rightarrow (?,q)$$

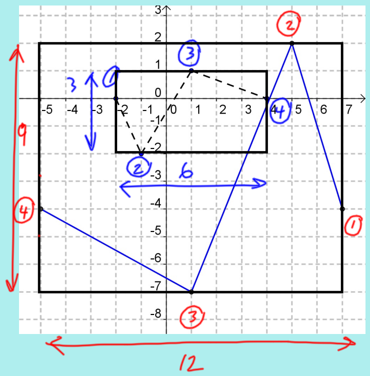
2. Consider the overall size (i.e., a box) of the graph, or a specific set of points on the graph to determine any stretches.

3. A pattern in the movement of points should show any reflections.

4. If all else fails, create up to four equations and solve for the four unknowns using ~~four~~ two points.

$$(x,y) \rightarrow \left(\frac{x}{k} + p,\ ay + q\right)$$

Ex.6 Determine the transformations shown and express in function notation.




$(-2,0) \rightarrow (7,-4)$

?

$\boxed{q=-4}$

$a<0, \; k<0$

height : $3 \rightarrow 9$

v. stretch by 3

$\boxed{a=-3}$

width : $6 \rightarrow 12$

h. stretch by 2

$\boxed{k=-\frac{1}{2}}$

p? $\left.\begin{array}{l}\text{h. reflect}\\ \text{h. stretch}\\ \text{h. shift}\end{array}\right\}$

$(x,y) \rightarrow \left(\frac{x}{k}+p, \; ay+q\right)$

$x \rightarrow \frac{x}{k}+p = 7$

$-2 \qquad 7$

$\frac{-2}{k}+p=7$

$\frac{-2}{-\frac{1}{2}}+p=7$

$4+p=7$

$\boxed{p=3}$

$(-1,-2) \rightarrow (5,2)$

$(-1,-2) \rightarrow \left(\frac{x}{k}+p, \; ay+q\right)$

$\rightarrow \left(\frac{-1}{-\frac{1}{2}}+3, \; -3(-2)-4\right)$

$\rightarrow (2+3, \; 6-4)$

$\rightarrow (5,2)$ ✓

$y=-3f\left[-\frac{1}{2}(x-3)\right]-4$ ✓