Inverse of a Function

March 8/2019

Recall: A relation is a set of ordered pairs.

The inverse of a relation can be found by interchanging the domain and range of the relation.

In other words, swap the x- and y-values for each point.

Ex.1 Determine the inverse of { (0,1), (3, 4), (2, -5) }.

The inverse is { (1, 0), (4, 3), (-5, 2) }.

(x, y)

input output

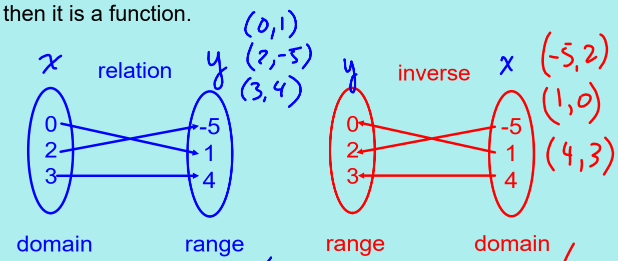
$x \rightarrow f(x) \rightarrow y$

input output

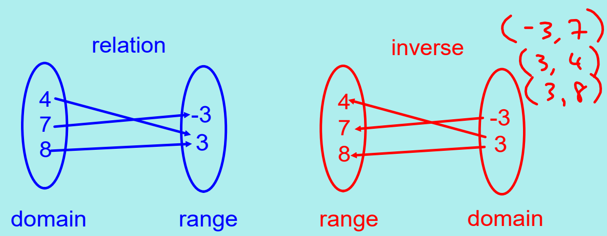
Feb 22-9:25 PM

A mapping diagram can be used to determine if a relation is a function.

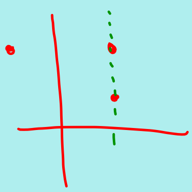
If there is only one arrow from each item in the domain, then it is a function.



relation is a function ✓ inverse is a function ✓



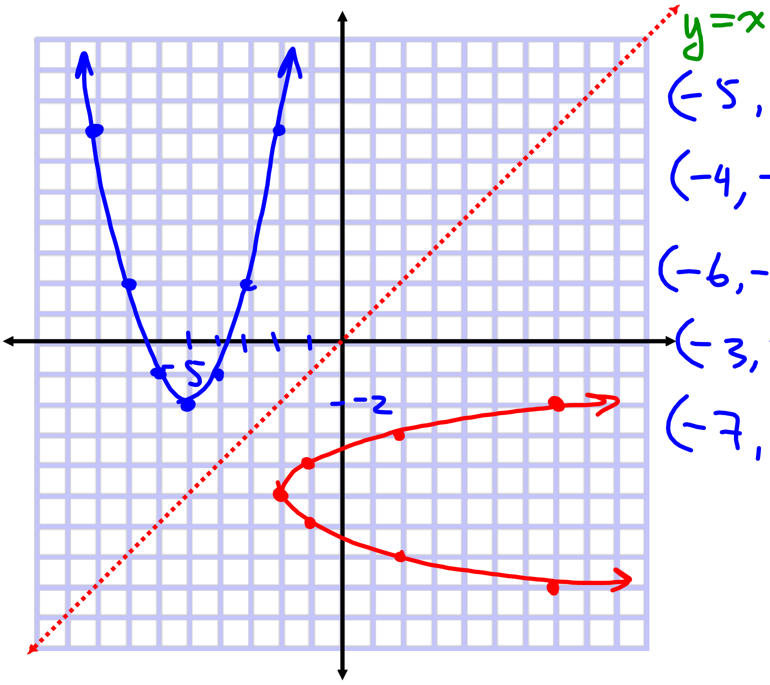
relation is a function inverse is NOT a function



Feb 22-8:37 PM

Graphically, swapping the x- and y-values is equivalent to reflecting the relation across the line y = x.

Ex.2 Find the inverse of $y=(x+5)^2-2$ graphically.



$(-5,-2) \to (-2,-5)$

$(-4,-1) \to (-1,-4)$

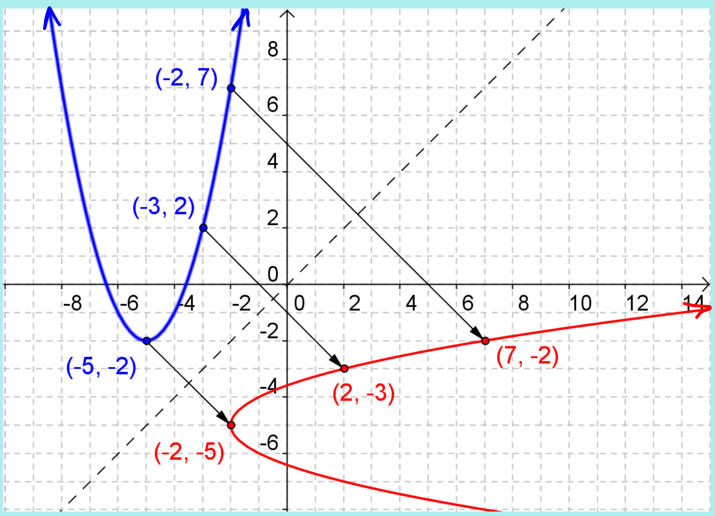
$(-6,-1) \to (-1,-6)$

$(-3,2) \to (2,-3)$

$(-7,2) \to (2,-7)$

Feb 22-10:16 PM

Find the inverse of $y=(x+5)^2-2$ graphically



Notice that the original points and the reflected (swapped) points are equidistant (equal distance) to the line y = x.

The inverse (red) fails the vertical line test, and is not a function.

Feb 23-9:20 PM

Recall: A function is a special type of relation where each element in the domain corresponds to a single value in the range.

For an inverse function, each value in the range corresponds to a single value in the domain.

If the inverse of the function, $f(x)$, is also a function, it is given the special designation of inverse function, $f^{-1}(x)$

Note: In the inverse notation, the "-1" is not an exponent!

For example:

$3^{-2} = \frac{1}{3^2}$ $\quad x^{-1} = \frac{1}{x}$ $\quad f^{-1}(x) \neq \frac{1}{f(x)}$

Feb 22-10:25 PM

Algebraically, swap the x- and y-variables, then rearrange the new equation for y.

If the original equation is in function notation, first change the function to y.

Ex.3 Find the inverse of $f(x) = 4x + 3$

rearrange for x

$y = 4x + 3$

$\frac{y-3}{4} = \frac{4x}{4}$

$x = \frac{y-3}{4}$

h (height)
t (time)
d (distance)
v (velocity)
c (cost)

c! swap x, y

$y = \frac{x-3}{4}$

inverse of $f(x) = 4x + 3$

Feb 22-10:21 PM

A function and its inverse <u>undo</u> each other. From the previous example,

$$f(x) = 4x + 3 \qquad f^{-1}(x) = \frac{x-3}{4} = \frac{1}{4}x - \frac{3}{4}$$

$$f(a) = b \rightarrow f^{-1}(b) = a$$

Ex.4 For each value, determine $y = f(x)$, then sub the y-value into $f^{-1}(x)$.

(a) x = 5 (b) x = -3 (c) x = 2.5

$f(5) = 4(5)+3$
$= 23$

$f^{-1}(23) = \frac{23-3}{4}$
$= \frac{20}{4}$
$= 5$

$f(2.5) = 4(2.5) + 3$
$= 10+3$
$= 13$

$f^{-1}(13) = \frac{13-3}{4}$
$= \frac{10}{4}$
$= 2.5$

Feb 22-9:04 PM

The inverse of a function might not be a function itself. It is possible to <u>restrict the domain</u> of the original function to force the inverse to also be a function.

Ex.5 Find the inverse of $f(x) = 3x^2 - 6$

$y = 3x^2 - 6$

swapping independent and dependent

$x =$ ______

* only when variables are x, y, do we swap them.

$y = 3x^2 - 6$

$\frac{y+6}{3} = \frac{3x^2}{3}$

$x^2 = \frac{y+6}{3}$

$x = \pm\sqrt{\frac{y+6}{3}}$

$\underbrace{y = \pm\sqrt{\frac{x+6}{3}}}$

Can be graphed with original function

swap x, y (swap independent/dependent)

$y = +\sqrt{\frac{1}{3}(x+6)}$ or $y = -\sqrt{\frac{1}{3}(x+6)}$

Feb 22-10:21 PM

$$C = \frac{5}{9}(F-32)$$

$$C(F) = \frac{5}{9}(F-32)$$

$$\frac{9}{5}C = F - 32$$

$$\frac{9}{5}C + 32 = F$$

$$\frac{9}{5}C + 32 = F(C)$$

Mar 18-2:07 PM

$$y = 3x^2 - 6$$

$$y + 6 = 3x^2$$

$$f(x) = \_\_\_\_$$

$$\frac{y+6}{3} = x^2$$

$$f^{-1}(x)$$

$$x = \pm\sqrt{\frac{y+6}{3}}$$

want y dependent, x independent

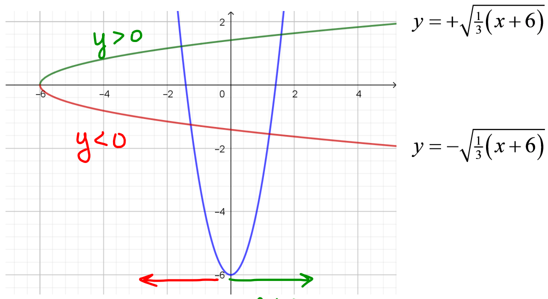
Swap x, y

$$y = \pm\sqrt{\frac{x+6}{3}}$$

Mar 8-3:14 PM

$f(x) = 3x^2 - 6$
original function

$y = \pm\sqrt{\frac{1}{3}(x+6)}$
inverse relation



$x<0 \qquad x>0$

$\downarrow$ swap x, y

$y<0 \qquad y>0$

$f(x) = 3x^2 - 6$

① restrict $D_f = \{x \in \mathbb{R} \mid x \geq 0\}$

$f^{-1}(x) = \sqrt{\frac{1}{3}(x+6)}$

OR

② restrict $D_f = \{x \in \mathbb{R} \mid x \leq 0\}$

$f^{-1}(x) = -\sqrt{\frac{1}{3}(x+6)}$

Feb 22-10:21 PM

Assigned Work:

p.215 # 1, 2, 3abci, 5odd, 8odd, 14(i)(iii)(v), 16(i)(iii)(v)

g g

5(g) $g(x) = \frac{5}{2}x - 4$

$y = \frac{5}{2}x - 4$

$\frac{2}{5}(y+4) = \left(\frac{5}{2}x\right) \times \frac{2}{5}$

$\frac{2}{5}(y+4) = x$

$y = \frac{2}{5}(x+4)$ swap x, y

OR

$y = \frac{2}{5}x + \frac{8}{5}$

Feb 10-10:23 PM

8(g) $h(x) = \frac{x-8}{4}$

$y = \frac{x-8}{4}$

$4y = x - 8$

$4y + 8 = x$

$4(y+2) = x$

swap x, y

$y = 4(x+2)$

$k(x) = 4(x+2)$

$y = 4(x+2)$

$\vdots$

since $f(f^{-1}(x)) = x$

$h(k(x)) = h(4(x+2))$

$= \frac{4(x+2) - 8}{4}$

$= \frac{4x + 8 - 8}{4}$

$= \frac{4x}{4}$

$= x$

Mar 18-2:34 PM

14 (i) $f(x) = x^2$, $x \geq 0$

(a) $y = x^2$

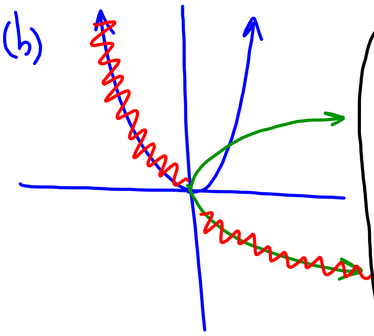
$\pm\sqrt{y} = x$

swap x, y $\quad y = \pm\sqrt{x}$

$y = \sqrt{x}$ ✓

$y = -\sqrt{x}$

(b)



(c) $D_f = \{x \in \mathbb{R} \mid x \geq 0\}$

$R_f = \{y \in \mathbb{R} \mid y \geq 0\}$

$D_{inv} = \{x \in \mathbb{R} \mid x \geq 0\}$

$R_{inv} = \{y \in \mathbb{R} \mid y \geq 0\}$

Mar 18-2:40 PM