Inverse of a Function

Recall: A relation is a set of ordered pairs.

The inverse of a relation can be found by interchanging the domain and range of the relation.

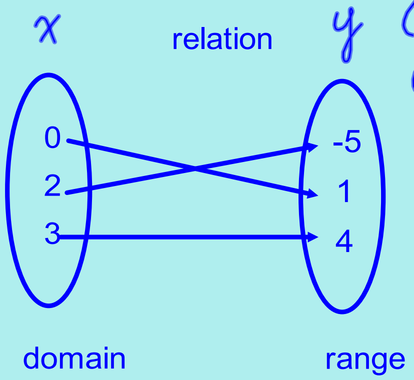
In other words, swap the x- and y-values for each point.

Ex.1 Determine the inverse of { (0,1), (3, 4), (2, -5) }. $f(x)$

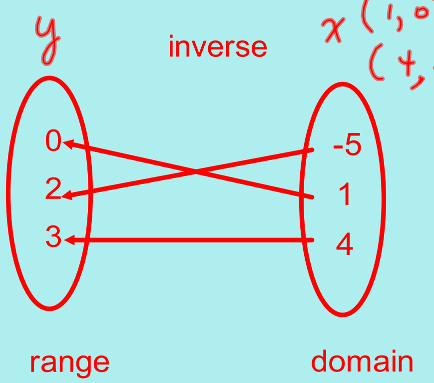
The inverse is { (1, 0), (4, 3), (-5, 2) }.

A mapping diagram can be used to determine if a relation is a function.

If there is only one arrow from each item in the domain, then it is a function.



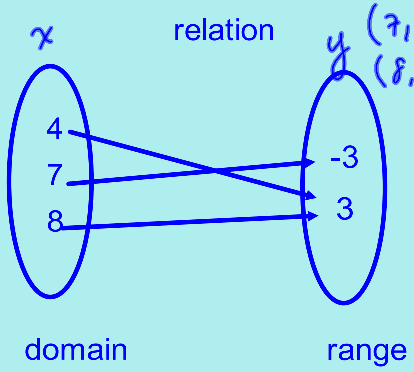
(0,1)
(2,-5)
(3,4)



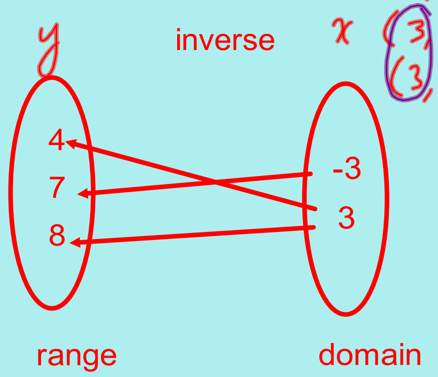
(-5,2)
(1,0)
(4,3)

relation is a function ✓

inverse is a function ✓



(4,3)
(7,-3)
(8,3)



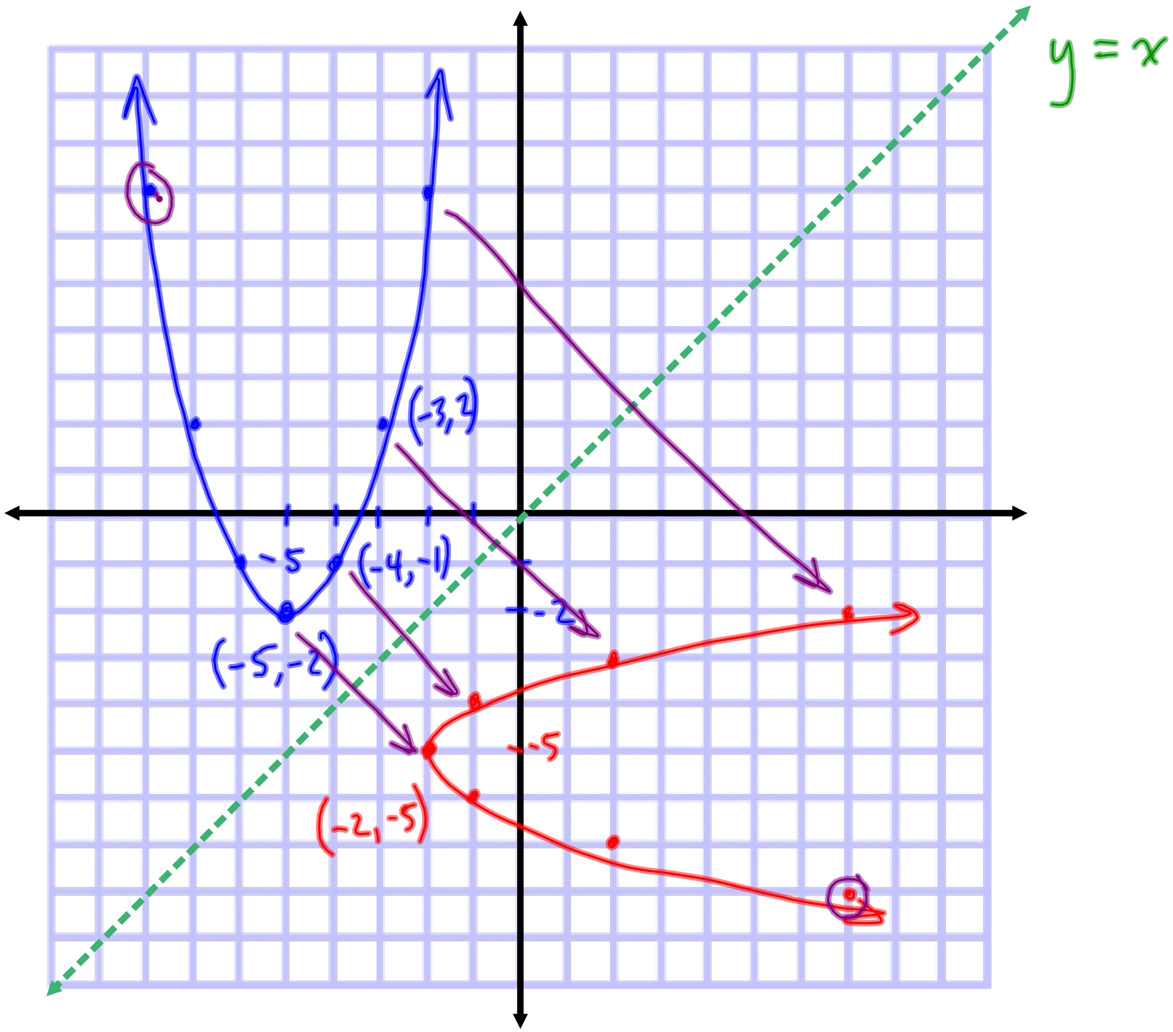
(-3,7)
(3,4)
(3,8)

relation is a function ✓

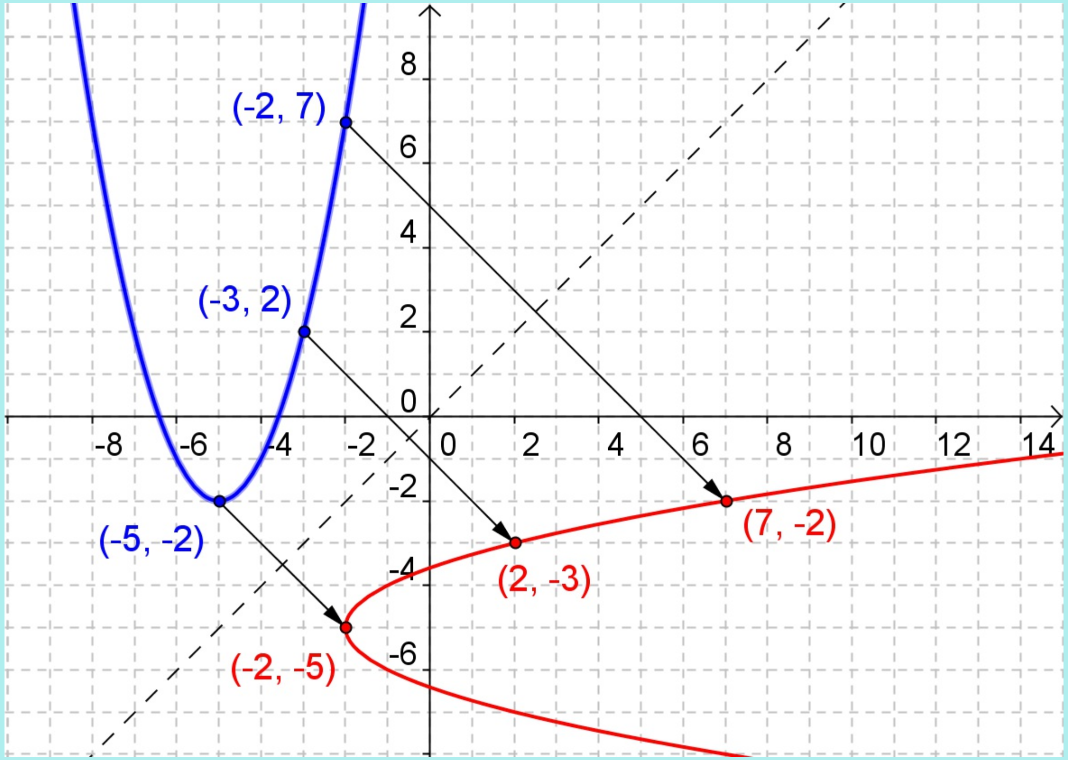
inverse is NOT a function ✗

Graphically, swapping the x- and y-values is equivalent to reflecting the relation across the line y = x.

Ex.2 Find the inverse of $y = 1(x+5)^2 - 2$ graphically.



Find the inverse of $y=(x+5)^2-2$ graphically



Notice that the original points and the reflected (swapped) points are equidistant (equal distance) to the line y = x.

The inverse (red) fails the vertical line test, and is <u>not</u> a function.

Recall: A <u>function</u> is a special type of relation where each element in the domain corresponds to a single value in the range.

For an <u>inverse function</u>, each value in the range $\underbrace{\text{corresponds to a single value}}_{\text{no repeated values.}}$ in the domain.

If the inverse of the function, $f(\overset{\text{input}}{x})$, is also a function, it is given the special designation of <u>inverse function</u>, $f^{-1}(x)$.

Note: In the inverse notation, the "-1" is not an exponent!

For example:

$$x^{-1} = \frac{1}{x} \qquad\qquad f^{-1}(x) \neq \frac{1}{f(x)}$$

Algebraically, swap the x- and y-variables, then rearrange the new equation for y.

If the original equation is in function notation, first change the function to y.

$y = f(x)$

Ex.3 Find the inverse of $f(x) = 4x + 3$

$y = 4x + 3$

for inverse, swap x, y

$x = 4y + 3$

$\frac{x-3}{4} = \frac{4y}{4}$

$y = \frac{x-3}{4}$

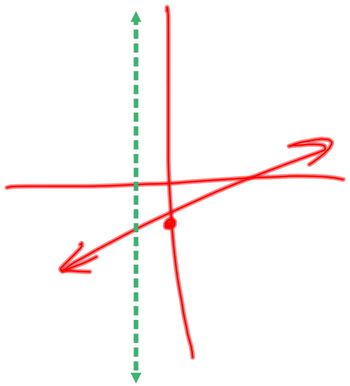
or

$y = \frac{x}{4} - \frac{3}{4}$

or

$y = \frac{1}{4}x - \frac{3}{4}$ is a function

$y = mx + b$



$f^{-1}(x) = \frac{1}{4}x - \frac{3}{4}$

A function and its inverse undo each other. From the previous example,

$$\underbrace{f(x) = 4x + 3}_{\text{output}} \qquad f^{-1}(x) = \frac{x-3}{4}$$

input

original input

Ex.4 For each value, determine $y = f(x)$, then sub the y-value into $f^{-1}(x)$.

(a) x = 5

$f(5) = 4(5)+3$

$= 23$

$f^{-1}(23) = \frac{23-3}{4}$

$= \frac{20}{4}$

$= 5$

(b) x = -3

$f(-3) = 4(-3)+3$

$= -12+3$

$= -9$

$f^{-1}(-9) = \frac{-9-3}{4}$

$= \frac{-12}{4}$

$= -3$

(c) x = 2.5

$f(2.5) = 4(2.5)+3$

$= 10+3$

$= 13$

$f^{-1}(13) = \frac{13-3}{4}$

$= \frac{10}{4}$

$= 2.5$

The inverse of a function might not be a function itself. It is possible to restrict the domain to force the inverse to be a function.

Ex.5 Find the inverse of $f(x) = 3x^2 - 6$

$y = 3x^2 - 6$

swap x, y

$x = 3y^2 - 6$

$\frac{x+6}{3} = \frac{3y^2}{3}$

$y^2 = \frac{x+6}{3}$

$y = \pm\sqrt{\frac{x+6}{3}}$

OR

$y = \pm\sqrt{\frac{1}{3}x + 2}$

$y = \pm\sqrt{\frac{1}{3}(x+6)}$

h. stretch of 3

left by 6

$x = 21$

$y = \pm\sqrt{\frac{1}{3}(21+6)}$

$= \pm\sqrt{\frac{1}{3}(27)}$

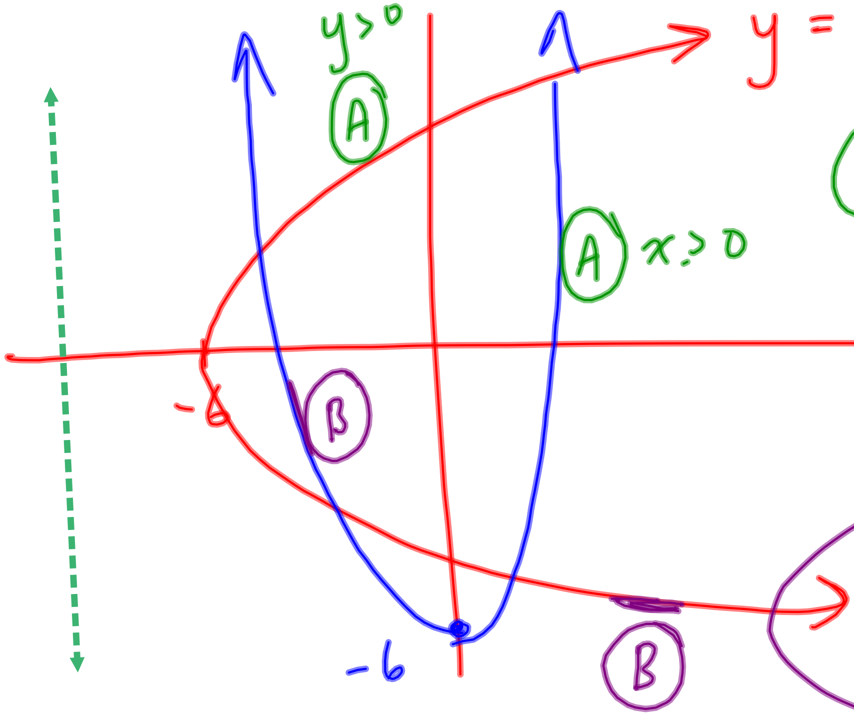
$= \pm\sqrt{9}$

$= \pm 3$

$(21, 3)$, $(21, -3)$

not a function

$y = \pm\sqrt{\frac{1}{3}(x+6)}$



$y = +\sqrt{\frac{1}{3}(x+6)}$

A : $D_f = \{x \in \mathbb{R} \mid x \geq 0\}$

$f^{-1}(x) = \sqrt{\frac{1}{3}(x+6)}$

$y = -\sqrt{\frac{1}{3}(x+6)}$

B $D_f = \{x \in \mathbb{R} \mid x \leq 0\}$

$f^{-1}(x) = -\sqrt{\frac{1}{3}(x+6)}$

Assigned Work:

p.215 # 1, 2, 3abci, 5odd, 8odd, 14(i)(iii)(v), 16(i)(iii)(v)