

Adding and Subtracting Rational Expressions

Recall these operations with fractions:

$$2 = \frac{2}{1}$$

$$14 = \frac{2 \cdot 7}{1}$$

Simplify each of the following

$$\frac{1}{2} + \frac{1}{5} \quad \text{LCD: } \underline{10}$$

$$= \frac{1 \times 5}{2 \times 5} + \frac{1 \times 2}{5 \times 2}$$

$$= \frac{5}{10} + \frac{2}{10}$$

$$= \frac{7}{10}$$
  

$$\frac{1 \times 7}{2 \times 7} + \frac{3}{14} \quad \text{LCD: } 2 \cdot 7 = 14$$

$$= \frac{7}{14} + \frac{3}{14}$$

$$= \frac{10}{14}$$

$$= \frac{5}{7}$$
  

$$\frac{5}{6} - \frac{3}{8} \quad \text{LCD: } \begin{matrix} = 24 \\ 2 \cdot 3 \cdot 2 \cdot 2 \\ 6 = 2 \cdot 3 \\ 8 = 2 \cdot 2 \cdot 2 \end{matrix}$$

$$= \frac{5 \times 4}{6 \times 4} - \frac{3 \times 3}{8 \times 3}$$

$$= \frac{20}{24} - \frac{9}{24}$$

$$= \frac{11}{24}$$
  

$$\frac{a}{b} + \frac{c}{d} \quad \text{LCD: } \begin{matrix} b = b \\ d = d \\ \text{LCD: } bd \end{matrix}$$

$$= \frac{a \cdot d}{b \cdot d} + \frac{c \cdot b}{d \cdot b}$$

$$= \frac{ad + bc}{bd}$$

Feb 12-9:14 PM

Adding and Subtracting Rational Expressions

Ex.1 Simplify and state any restrictions

$$(a) \quad \frac{x}{x+1} + \frac{3x}{x+2} \quad \text{LCD} = (x+1)(x+2)$$

$$= \frac{x(x+2)}{(x+1)(x+2)} + \frac{3x(x+1)}{(x+2)(x+1)}$$

$$= \frac{(x^2 + 2x) + (3x^2 + 3x)}{(x+1)(x+2)}$$

$$= \frac{4x^2 + 5x}{(x+1)(x+2)}$$

$$= \frac{x(4x+5)}{(x+1)(x+2)}, \quad x \neq -1, x \neq -2$$

Mar 20-11:09 PM

Steps:

1. Factor the denominator.
2. Identify restrictions (where is denominator = 0 ?).
3. Find the lowest common denominator, LCD
4. Express each rational expression with the same LCD
5. Add/subtract the terms in the numerator, keep the LCD (factored form) as your denominator.
6. Factor the numerator, if possible, and simplify.

Mar 21-10:21 AM

$$(b) \frac{x+2}{x-2} - \frac{x}{(x-2)^2}$$

$$(x-2)^2 = (x-2)(x-2)$$

$$= \frac{(x+2)(x-2)}{(x-2)(x-2)} - \frac{x}{(x-2)^2}$$

$$LCD : (x-2)(x-2) = (x-2)^2$$

$$= \frac{(x^2-4) - (x)}{(x-2)^2}$$

$$= \frac{x^2 - x - 4}{(x-2)^2}, \quad x \neq 2$$

factor?

S - 1  
P - 4  
I? X

Mar 20-11:08 PM

$$(c) \frac{2}{x^2+x} + \frac{x}{x^2-1} \quad \text{LCD: } x(x+1)(x-1)$$

$$= \frac{2}{x(x+1)} + \frac{x}{(x-1)(x+1)}$$

$$= \frac{2(x-1)}{x(x+1)(x-1)} + \frac{x(x)}{(x-1)(x+1)(x)}$$

$$= \frac{(2x-2) + (x^2)}{x(x-1)(x+1)} \quad \begin{array}{l} S \ 2 \\ P \ -2 \\ I \ \times \end{array}$$

$$= \frac{x^2 + 2x - 2}{x(x-1)(x+1)}, \quad x \neq 0, x \neq 1, x \neq -1$$

OR  
 $x \neq 0, 1, -1$

OR

$$x \neq 0, \pm 1$$

Mar 20-11:23 PM

$$(d) \frac{2x-6}{3x+6} + \frac{x-3}{x^2+5x+6}$$

$$\begin{array}{l} S \ 5 \\ P \ 6 \\ I \ 2, 3 \end{array}$$

$$= \frac{2(x-3)}{3(x+2)} + \frac{x-3}{(x+2)(x+3)}$$

$$x \neq -2$$

$$x \neq -3$$

$$= \frac{2(x-3)(x+3)}{3(x+2)(x+3)} + \frac{(x-3)(?)}{(x+2)(x+3)(?)}$$

$$\text{LCD: } 3(x+2)(x+3)$$

$$= \frac{2(x^2-9) + (3x-9)}{3(x+2)(x+3)}$$

$$1 \times 54$$

$$2 \times 27$$

$$3 \times 18$$

$$6 \times 9$$

$$= \frac{2x^2 + 3x - 27}{3(x+2)(x+3)}$$

$$S \ 3$$

$$P \ -54$$

$$I \ 9, -6$$

$$= \frac{(2x+9)(x-3)}{3(x+2)(x+3)}, \quad x \neq -2, -3$$

$$2x^2 + 9x - 6x - 27$$

$$= x(2x+9) - 3(2x+9)$$

$$= (2x+9)(x-3)$$

Mar 20-11:23 PM

Assigned Work:

p.58 # 1c, 4c, 5g, 6ac, 10

p.67 # 5c, 6ace, 8ace

p.58

$$5g) \frac{5x-1}{5} + \frac{1}{1} - \frac{4x-3}{6} \quad \text{LCD: } 30$$

$$= \frac{(5x-1) \cdot \frac{6}{6}}{5 \cdot \frac{6}{6}} + \frac{1 \cdot \frac{30}{30}}{1 \cdot \frac{30}{30}} - \frac{(4x-3) \cdot \frac{5}{5}}{\frac{6}{6} \cdot \frac{5}{5}}$$

$$= \frac{(30x-6) + 30 - (20x-15)}{30}$$

$$= \frac{30x - 6 + 30 - 20x + 15}{30}$$

$$= \frac{10x + 39}{30}$$

Mar 20-11:27 PM

p.58

$$6(c) \quad \frac{a-2}{2a-3} + \frac{a+3}{3-2a} \quad \begin{array}{l} 3-2a \\ = -2a+3 \\ = -1(2a-3) \end{array}$$

$$= \frac{a-2}{2a-3} + \frac{a+3}{-(2a-3)}$$

$$= \frac{a-2}{2a-3} - \frac{a+3}{2a-3}$$

$$= \frac{(a-2) - (a+3)}{2a-3}$$

$$= \frac{-5}{2a-3}, \quad a \neq \frac{3}{2}$$

$$2a-3=0$$

$$2a=3$$

$$a = \frac{3}{2}$$

Mar 26-2:09 PM

10.

$$p = \frac{6x^2 + 5x + 1}{2x + 1}$$

$$q = \frac{4x^2 - 4x - 3}{2x + 1}$$

(a)  $p(2x+1) = 6x^2 + 5x + 1$   
 $p = \frac{6x^2 + 5x + 1}{2x + 1}$

(b)  $q = \frac{4x^2 - 4x - 3}{2x + 1}$

(c)  $q - p = \frac{4x^2 - 4x - 3}{2x + 1} - \frac{6x^2 + 5x + 1}{2x + 1}$   
 $= \frac{(4x^2 - 4x - 3) - (6x^2 + 5x + 1)}{2x + 1}$   
 $= \frac{-2x^2 - 9x - 4}{2x + 1}$   
 $= -\frac{2x^2 + 9x + 4}{2x + 1}$   
 $= -\frac{(2x+1)(x+4)}{2x+1}$   
 $= -(x+4), x \neq -\frac{1}{2}$   
 $= -x - 4$

*S 9  
P 8  
I 1, 8*

*$2x^2 + 9x + 4$   
 $= x(2x+1) + 4(2x+1)$   
 $= (2x+1)(x+4)$*

Mar 26-2:13 PM

p. 67

6(a)  $\frac{2}{x+1} + \frac{3}{x+2}$

$$= \frac{2(x+2)}{(x+1)(x+2)} + \frac{3(x+1)}{(x+2)(x+1)}$$

$$= \frac{(2x+4) + (3x+3)}{(x+1)(x+2)}$$

$$= \frac{5x+7}{(x+1)(x+2)}, x \neq -1, -2$$

Mar 26-2:21 PM

$$\begin{aligned} 8(a) \quad & \frac{2}{x+3} + \frac{3}{x^2+5x+6} \\ & = \frac{2}{x+3} + \frac{3}{(x+2)(x+3)} \\ & = \frac{2(x+2) + 3}{(x+2)(x+3)} \\ & = \frac{2x+7}{(x+2)(x+3)}, x \neq -2, -3 \end{aligned}$$

Mar 26-2:22 PM